- Derivate e integrali. In genere, date le generiche $y(t)$ e $x(t)$,

$$y = \frac{dx}{dt} \quad \longleftrightarrow \quad dx = ydt \to \Delta x|_{t_1}^{t_2} = \int_{t_1}^{t_2} y(t)\, dt\,.$$

- Basi di cinematica:

$$\begin{aligned} \vec{v} &= \frac{d\vec{s}}{dt} \quad (\text{anche } \frac{d\vec{r}}{dt}) \\ \vec{a} &= \frac{d\vec{v}}{dt}\,, \end{aligned}$$

con $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$, etc.

- Equazione parametrica cerchio. Velocità e accelerazione (anche delle componenti).

$$\begin{aligned} \vec{r}(t) &= \begin{cases} x(t) = R\cos\alpha(t) = R\cos\omega t \\ y(t) = R\sin\alpha(t) = R\sin\omega t \end{cases} \\ v(t) &= \left|\frac{d\vec{r}}{dt}\right|\,; \quad a(t) = \left|\frac{d\vec{v}}{dt}\right| \end{aligned}$$

Periodo: $t = T \implies \alpha(t) = 2\pi$.
Frequenza (ν): giri/s. Vel. ang (ω): rad/s.
$v(t)$ a $a(t)$ costanti in moto circ. uniforme.

- Dinamica

$$\begin{aligned} \vec{F} &= m\,\vec{a} = \frac{d\vec{p}}{dt} \qquad [\vec{p} = m\vec{v}] \\ \vec{F}_B(A) &= -\vec{F}_A(B) \\ \vec{F}_{tot} &= \sum_i \vec{F}_i\,. \end{aligned}$$

- Forze (un inventario)

$$\begin{aligned} F_G &= -G\,\frac{m_1\,m_2}{d^2} \qquad \left[G = 6.67\times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kgs}^2}\right] \\ &= -m_2 g \qquad [\text{se } m_1 = M_T \text{ e } d = R_T] \\ F_C &= k_0\,\frac{q_1\,q_2}{d^2} \qquad \left[k_0 = 9\times 10^9\,\mathrm{N\,m^2/C^2}\right] \\ \vec{F}_L &= q\cdot\vec{v}\wedge\vec{B} \\ F_{el} &= -k\,x \\ \vec{F}_{A_d} &= -\mu_d\,F_N\,\hat{v} \\ \vec{F}A_v &= -\beta\vec{v} \\ F_{A_s} &\leq \mu_s\,F_N \end{aligned}$$

$\vec{T}$? $\vec{F}_{A_s}$? $\Rightarrow$ ‘vincoli’.

- Oscillatore armonico:

$$\begin{aligned} \frac{d^2}{dt^2}\,z(t) &= -\omega^2\,z(t) \iff z(t) = A\,\cos(\omega t + \phi)\,. \\ \frac{d^2x}{d\,t^2} &= -\frac{k}{m}\,x\,; \\ \frac{d^2\theta}{dt^2} &\approx -\frac{g}{l}\,\theta\,; \\ \frac{d^2Q}{dt^2} &= -\frac{1}{LC}\,Q\,; \end{aligned}$$

- Trasformazioni di velocità:

$$\vec{r}_{O'}(P) = \vec{r}_{O'}(O) + \vec{r}_O(P)\,.$$

- Lavoro, energia cinetica e potenziale. Potenza.

$$\begin{aligned} dL &= \vec{F}\cdot d\vec{s} \\ L|_A^B = \int_A^B dL &= \Delta\left(\frac{1}{2}mv^2\right)\Big|_A^B \\ [&= -\Delta E_p|_A^B \quad \text{solo } \vec{F} \text{ cons.}] \\ P &= \frac{dL}{dt}. \end{aligned}$$

- Centro di massa (’baricentro’):

$$\begin{aligned} \vec{r}_{CM} &= \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \\ \vec{F}_{tot}^{ext} &= \frac{d\vec{P}_{tot}}{dt} = \frac{d}{dt}\sum_i \vec{p}_i. \end{aligned}$$

- Urti elastici: $\to$ conservano energia meccanica.

- Termometria (qui Q indica quantità di calore):

$$\begin{aligned} Q &= C\,\Delta T \\ \sum_i Q_i &= 0 \qquad (\text{sistema isolato}) \\ 1\,\mathrm{cal} &= 4.184\,\mathrm{J} \\ \lambda_{H_2O} &= 80\,\mathrm{cal/g}\ (\text{fusione}) \\ &= 540\,\mathrm{cal/g}\ (\text{ebolliz.}) \end{aligned}$$

- Potenziale e campo elettrico:

$$\begin{aligned} \Delta E_p|_A^B &= q\,\Delta V|_A^B = q\,\Delta V_{AB} \\ F_C^q(\vec{r}) &= q\,\vec{E}(\vec{r}) \\ \Delta V|_A^B &= -\int_A^B \vec{E}\cdot d\vec{s} \\ \vec{E}(r) &= \sum_i E_i(r) = \sum_i \frac{k_0\,q_i}{|\vec{r}-\vec{r}_i|^3}\,(\vec{r}-\vec{r}_i) \\ V(r) &= \sum_i V_i(r) = \sum_i \frac{k_0\,q_i}{|\vec{r}-\vec{r}_i|}\,. \end{aligned}$$

- Circuiti

$$\begin{aligned} I_{A\to B}\left[\equiv \frac{dq}{dt}\right] &= \frac{V_A - V_B}{R} \\ dL &= dq\,\Delta V \\ \frac{dL}{dt} &= \frac{dq}{dt}\,\Delta V \\ \sum_i I_i &= 0 \quad [\text{nodi}] \\ \sum_i \Delta V_i &= 0 \quad [\text{maglie}] \\ Q &= C\,V\,. \end{aligned}$$

differenze di potenziale.
Resistenze in parallelo: stessa differenza di potenziale $\rightarrow$ si sommano le correnti.
Condensatori serie/parallelo: $\rightarrow$ regole opposte a quelle delle resistenze.

- Andamenti esponenziali

$$\begin{aligned} \frac{dx}{dt} &= \alpha\,(x_f - x) \\ \Rightarrow \quad x(t) &= x_f + (x_0 - x_f)\,e^{-t/\tau} \\ & \quad [\tau = 1/\alpha]\,. \end{aligned}$$

Condensatore-resistenza: $\alpha = 1/(RC)$.
Termalizzazione: $\alpha = \eta/(cM)$.
Vel. limite (attrito tipo $-\beta v$): $\alpha = \beta/m$.

- Circuito con resistenza (R), capacità (C) e induttanza (L). Tabella di analogie:

$$\begin{aligned} x &\leftrightarrow Q \\ v &\leftrightarrow I \\ a &\leftrightarrow \frac{dI}{dt} \\ m &\leftrightarrow L \\ k &\leftrightarrow \frac{1}{C} \\ \beta &\leftrightarrow R \\ \frac{1}{2}\,m\,v^2 &\leftrightarrow \frac{1}{2}\,L\,I^2 \\ \frac{1}{2}\,k\,x^2 &\leftrightarrow \frac{1}{2}\,\frac{1}{C}\,Q^2 \\ \beta\,v^2 &\leftrightarrow R\,I^2 \\ \sqrt{\frac{k}{m}} &\leftrightarrow \frac{1}{\sqrt{L\,C}} \end{aligned}$$

- Oscillatore smorzato:

$$\begin{aligned} \frac{d^2z}{dt^2} + \gamma\,\frac{dz}{dt} + \omega_0^2\,z &= 0 \\ \gamma &= \frac{\beta}{m} \ \text{ ovvero } \ \frac{R}{L} \\ \omega_0^2 &= \frac{k}{m} \ \text{ ovvero } \ \frac{1}{LC} \\ \left(\frac{\gamma}{2}\right)^2 - \omega_0^2 &> 0 \ \rightarrow \text{sovrasmorzata} \\ \left(\frac{\gamma}{2}\right)^2 - \omega_0^2 &= 0 \ \rightarrow \text{critica} \\ \left(\frac{\gamma}{2}\right)^2 - \omega_0^2 = -\omega_1^2 &< 0 \ \rightarrow \text{sottosmorzata} \end{aligned}$$

Caso sottosmorzato con $z(0) = z_0$ e $\dot{z}(0) = 0$:

$$\begin{aligned} \Rightarrow z(t) &= \frac{z_0}{\cos\varphi}\,e^{-t/\tau}\,\cos(\omega_1 t + \varphi) \\ & \quad [\tau = 2/\gamma] \\ & \quad Q = \frac{\omega_0}{\gamma}\,. \end{aligned}$$

$$\begin{aligned} \frac{d^2z}{dt^2} + \gamma\,\frac{dz}{dt} + \omega_0^2\,z &= \eta_0\,\cos\omega t\,, \\ \eta_0 &= f_0/m \ \text{ ovvero } \ f_0/L \\ \Rightarrow \quad z_0 &= \frac{\eta_0}{\omega_0^2 - \omega^2 + j\omega\,\gamma}\,. \end{aligned}$$

- Corpo rigido (in particolare, ruotante intorno ad un asse fisso):

$$\begin{aligned} \vec{M} &= \vec{r} \wedge \vec{F} \qquad \left[\vec{M}_{tot} = \sum_i \vec{M}_i\right] \\ \vec{L} &= \vec{r} \wedge \vec{p} \qquad \left[\vec{L}_{tot} = \sum_i \vec{L}_i\right] \\ I &= \sum_i m_i r_i^2 \ \ [= \sum_i I_i\ ] \ \rightarrow \text{“}\int \mathrm{d}I\,\text{”} \end{aligned}$$

$$\begin{aligned} x &\leftrightarrow \theta \\ v = \frac{dx}{dt} &\leftrightarrow \omega = \frac{d\theta}{dt} \\ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} &\leftrightarrow \dot{\omega} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \\ m &\leftrightarrow I \\ p = mv &\leftrightarrow L = I\omega \\ F = \frac{dp}{dt} = ma &\leftrightarrow M = \frac{dL}{dt} = I\dot{\omega} \\ \ldots\ldots\ldots\ldots &\leftrightarrow \ldots\ldots\ldots\ldots \end{aligned}$$

In particolare, $M^{(ext)} = 0 \ \Rightarrow L = cost$

- Fluidi (idrostatica):

$$\begin{aligned} P &= \frac{\vec{F}\cdot\vec{n}}{A} \\ \frac{dP}{dz} &= \rho\,g \qquad (z > 0 \text{ verso il basso}) \\ \Delta P &\rightarrow \text{si trasmette a tutto il fluido} \\ F_A &= \rho_f\,V_{f.s.}\,g \qquad (\text{verso l'alto})\,. \end{aligned}$$