Measurements, uncertainties and probabilistic inference/forecasting

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"Probability is good sense reduced to a calculus"
(S. Laplace)

Introducing the logic of uncertainty

- No collection of formulae.

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- Try to build up a consistent theory
  that can be used for a broad range of applications.
  - Avoid unneeded '*principles*'...
    whose results will *possibly* be reobtained
    as *approximations under well stated conditions*.

Please be patient

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"...today I'll learn to read,

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"...today I'll learn to read,

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- No rush to get formulae

→ If you understand the basic reasoning
you can derive many formulae by yourself' !

Observation $\rightarrow$ value of a quantity

scale reading $\xrightarrow[\text{given } g,\ k,\ \text{“etc.”}\ldots]{}$ m

Observations $\rightarrow$ hypotheses

This problem occurs not only "determining" *the* value of a physical quantity.

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(But logically no substantial difference.)

Human ancestral problem

? ? ?

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? ? ?

→ Chase?

→ Run away?

Observation $\rightarrow$ hypotheses

Dependence from the context

Chase o Run away?

Dependence from the context

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. . . or simply stay quite

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if it is a mold in a museum,

or an artificial track in a school garden,

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Dependence from the context

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(. . . or we are just sated tourists, with no interest in chasing, well protected inside our safari minibus ☺)

Contemporary anthropology (and technology)

? ? ?

Effect and possible causes

Effect: car broken down

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- Causes:
  - no gasoline
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  - he has his own ideas about most likely causes.
- Action: balance between probability of the several hypotheses, costs and times.

Dipendenza dal contesto

Scambio di whatsapp con vecchio amico meccanico
(24 aprile 2015)

> "Sto per andare ai Laboratori di Frascati.
> Dovevo stare lì alle 6, ma mi hanno mandato una mail dicendo che hanno problemi con l'acceleratore e posso arrivare alle 8."

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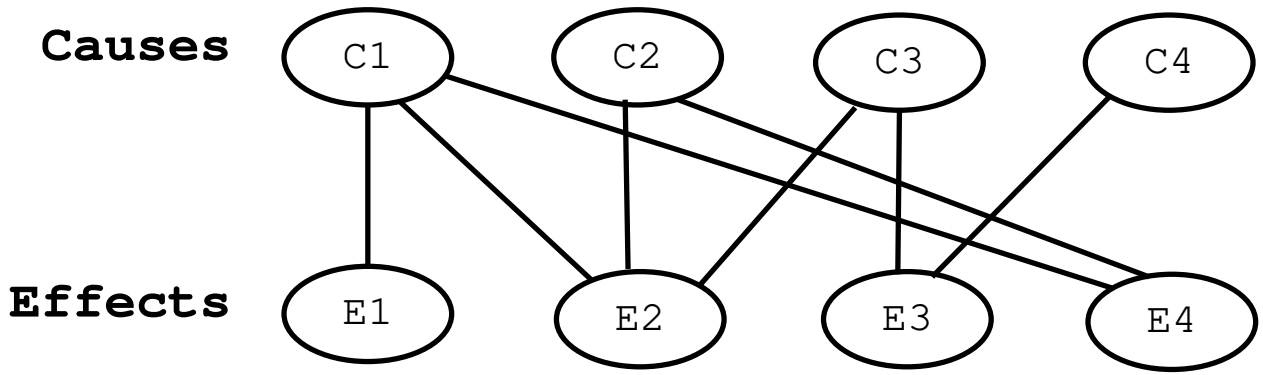
"Forse sarà il filo che non scorre bene nella guaina."

Causes → effects

The same *apparent* cause might produce several,different effects



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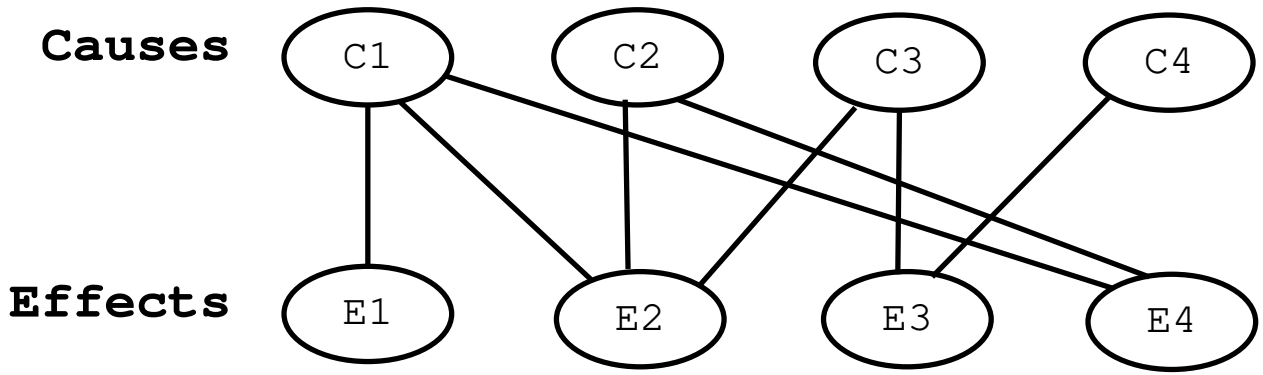

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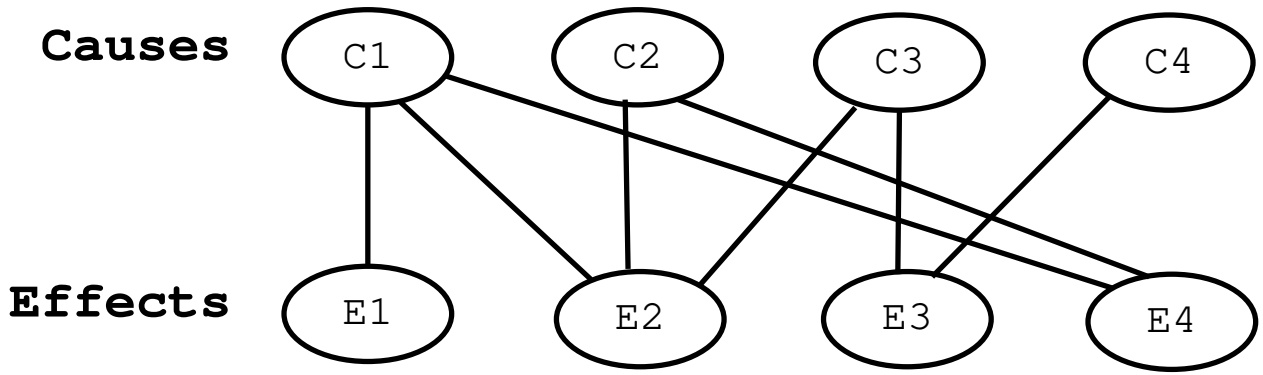

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$$\mathbf{E_2} \Rightarrow \{C_1, C_2, C_3\}?$$

The "essential problem" of the Sciences

"Now, these problems are classified as *probability of causes*, and are most interesting of all for their scientific applications. I play at *écarté* with a gentleman whom I know to be perfectly honest. What is the chance that he turns up the king? It is 1/8. This is a problem of the probability of effects.

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I play with a gentleman whom I do not know. He has dealt ten times, and he has turned the king up six times. What is the chance that he is a sharper? This is a problem in the probability of causes. It may be said that it is the essential problem of the experimental method."

(H. Poincaré – *Science and Hypothesis*)

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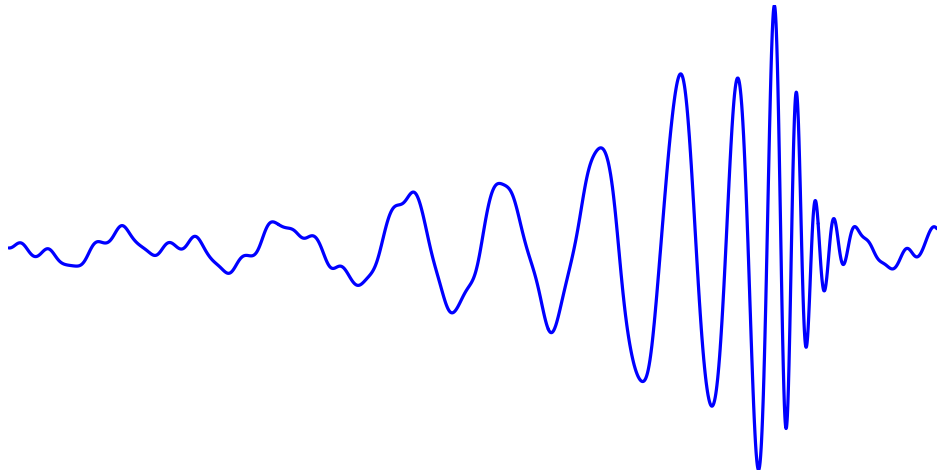
> (H. Poincaré – *Science and Hypothesis*)

Why we (or most of us) have not been taught how to tackle this kind of problems?

Who has done this ‘scribble’?



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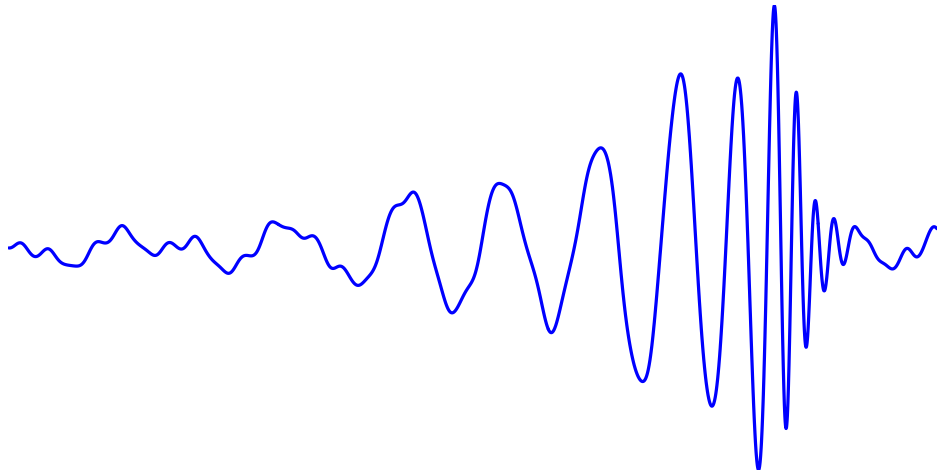

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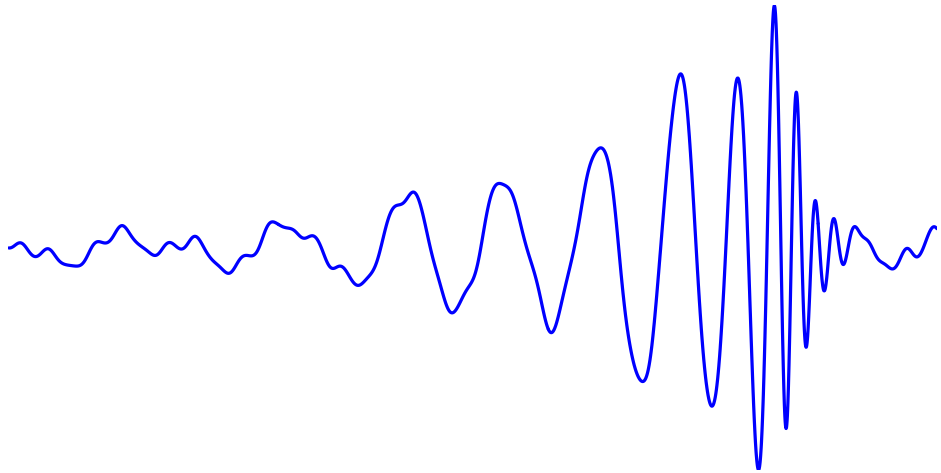

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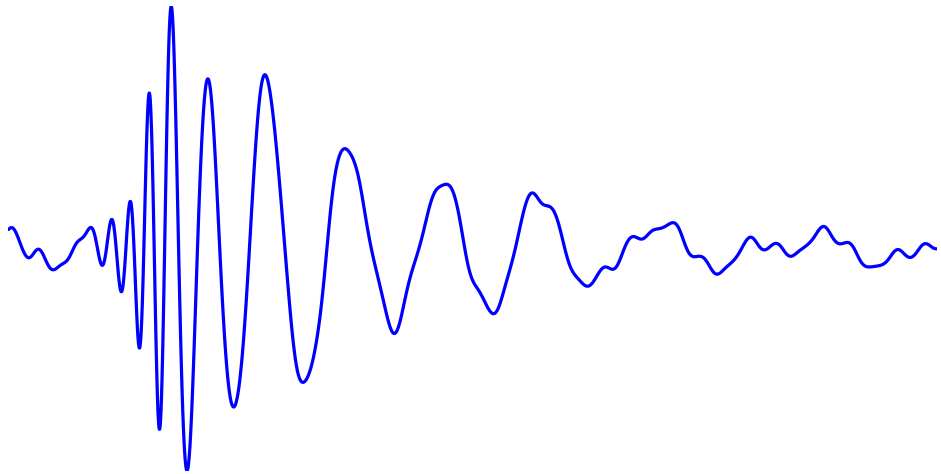

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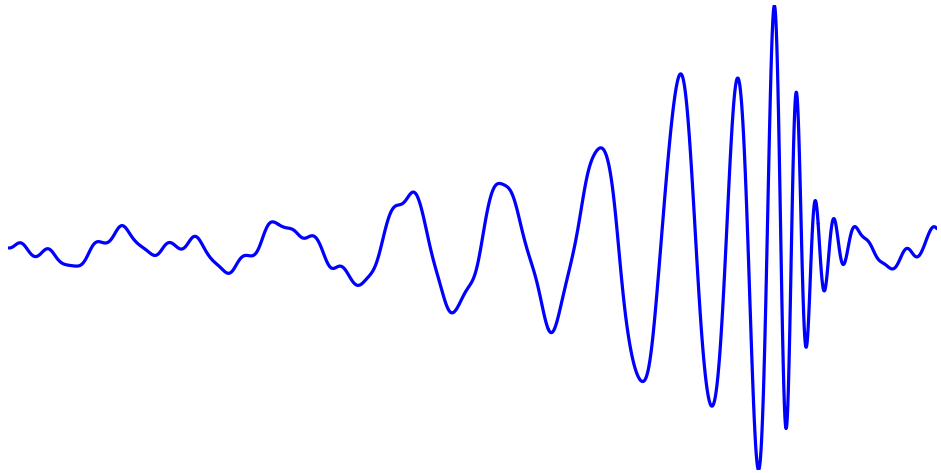

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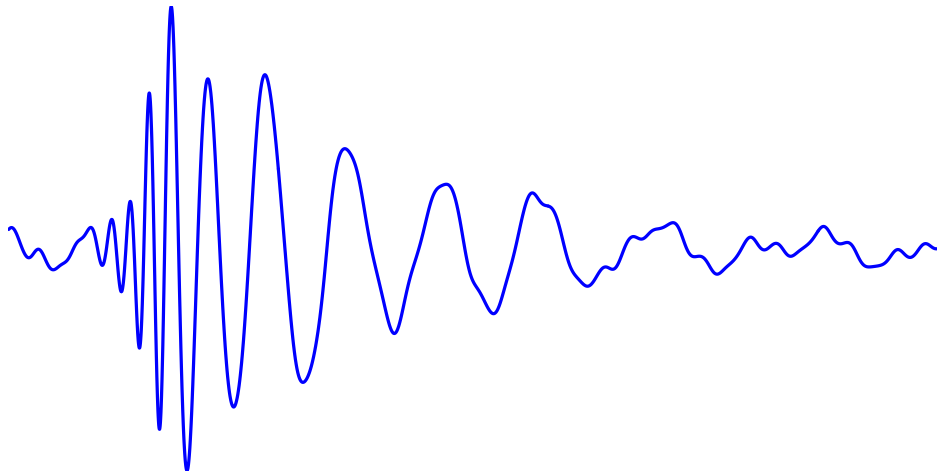
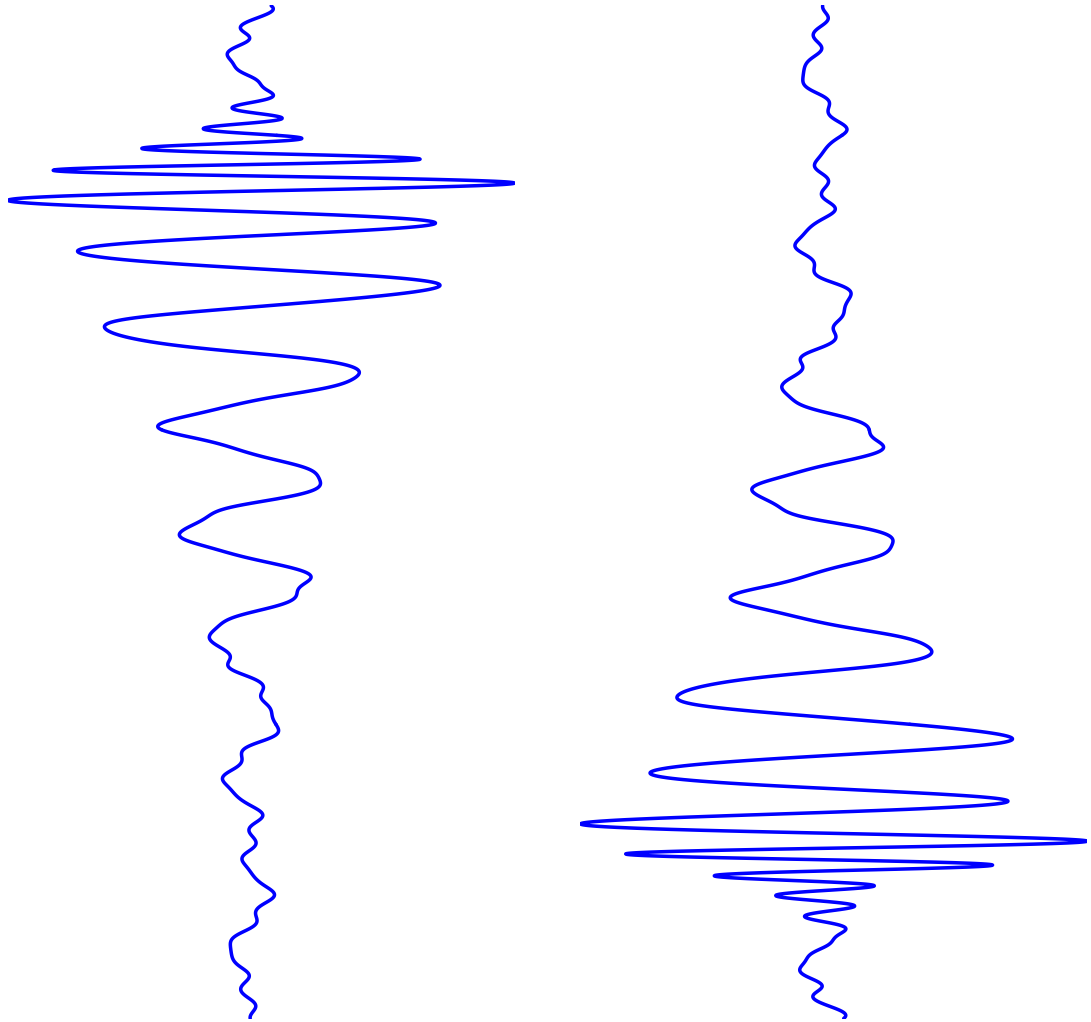


???

- Cardiogram?
- Signature?
- Sound?
- Earthquake?

Let's change orientation

(pure despair...)



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Such an information, lacking details about

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Distrust the

Dogma of the Immaculate Observation

Context + further details

Things change completely when we get informed that

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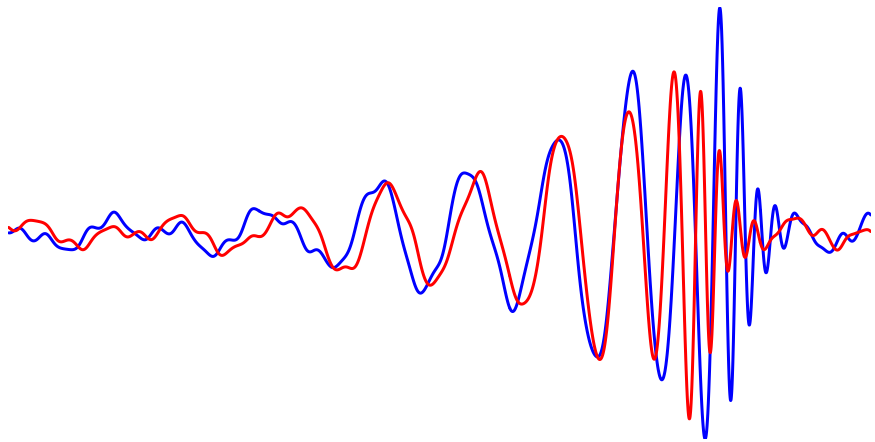

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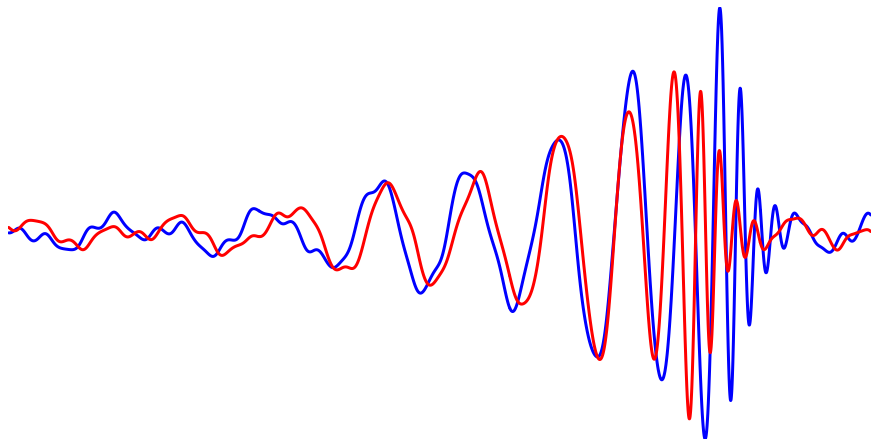

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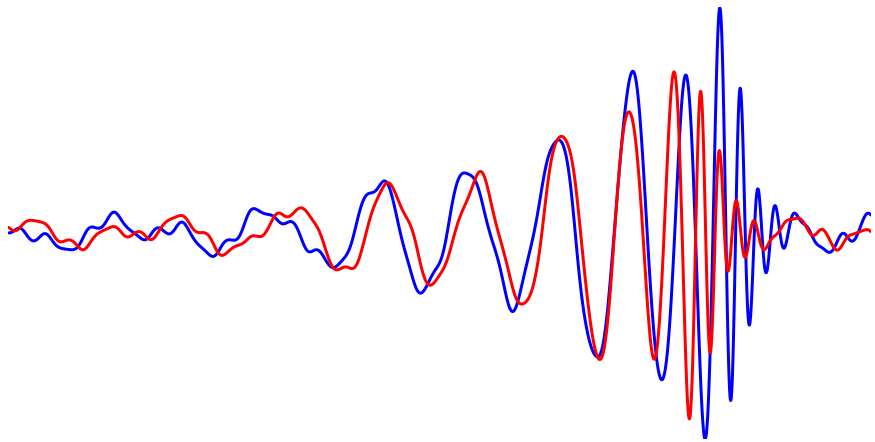

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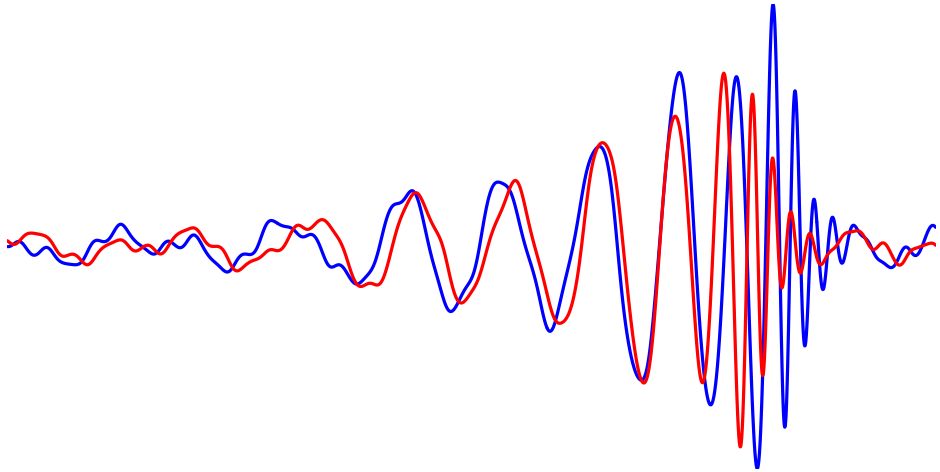
  [We tend to believe what trusted people believe]

Effect $\rightarrow$ cause



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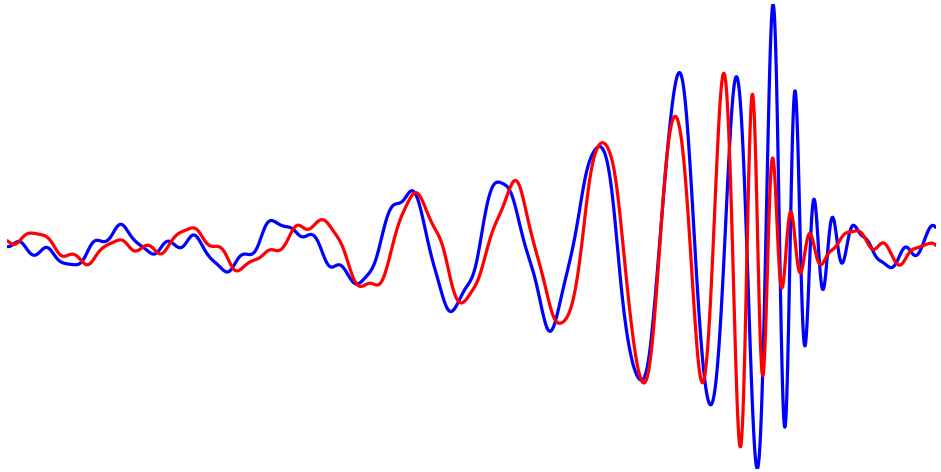

? ? ?

- The expected 'signer'?

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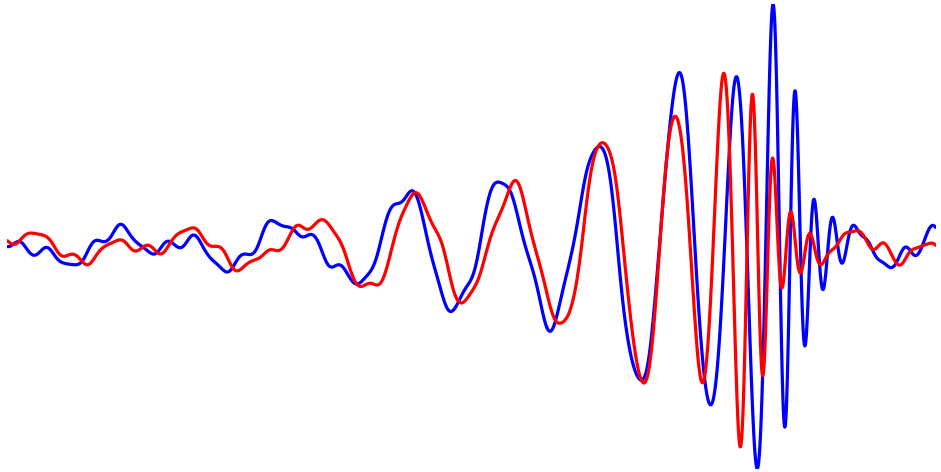

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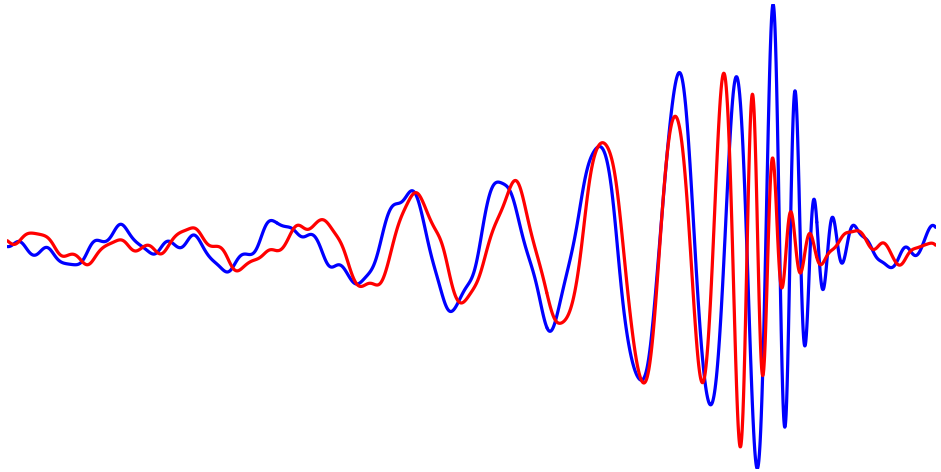

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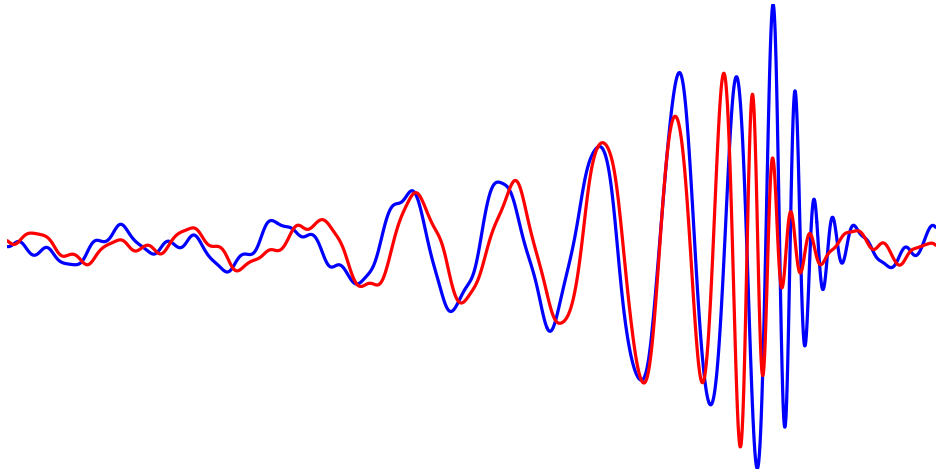

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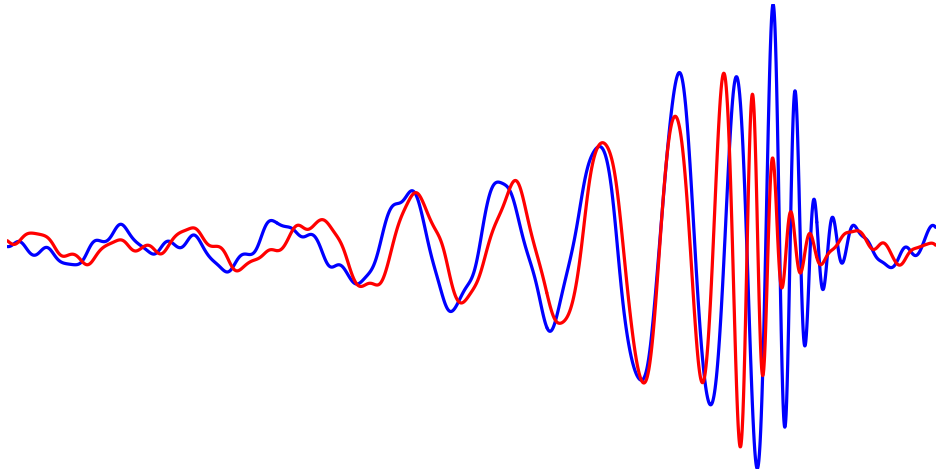

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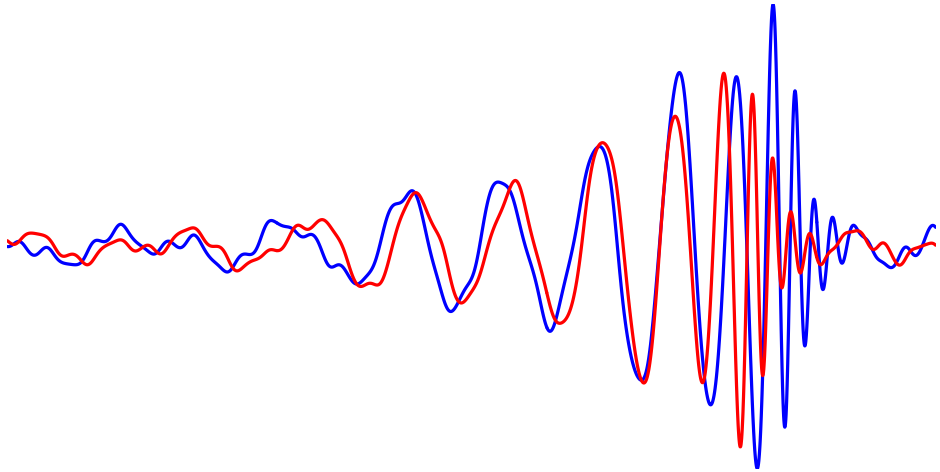

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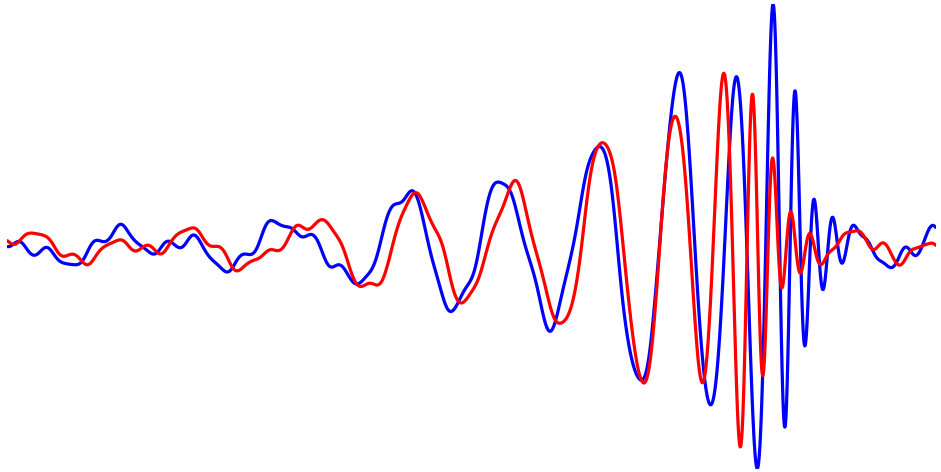

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(The last two causes are not just amenities!)

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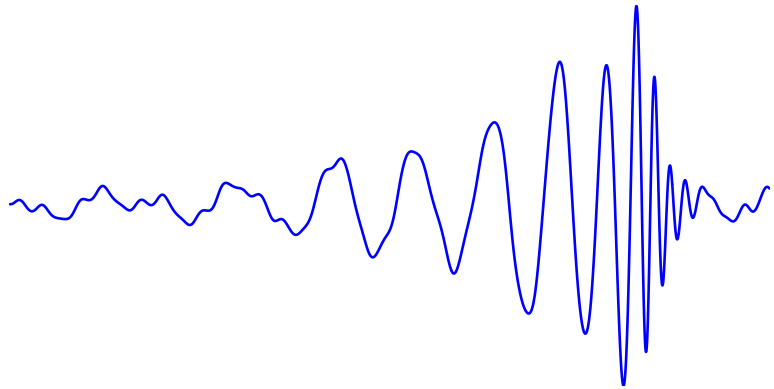
On the basis of the best knowledges about the possible causes



$\Rightarrow$ Gravitational wave

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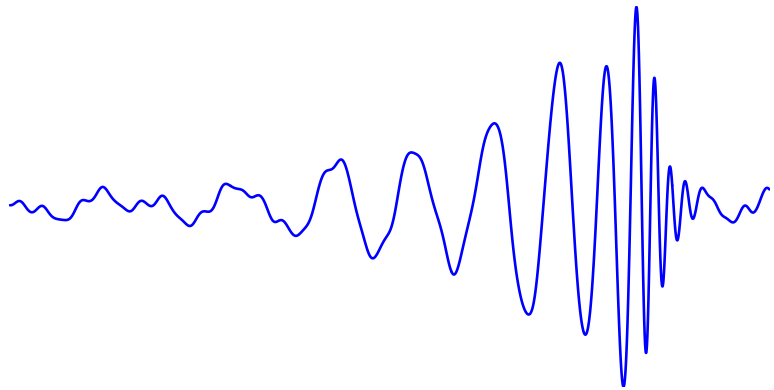

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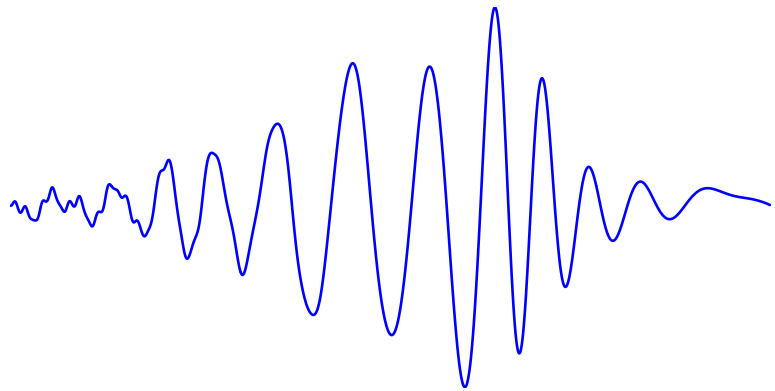
And if instead would have been this other 'scribble'



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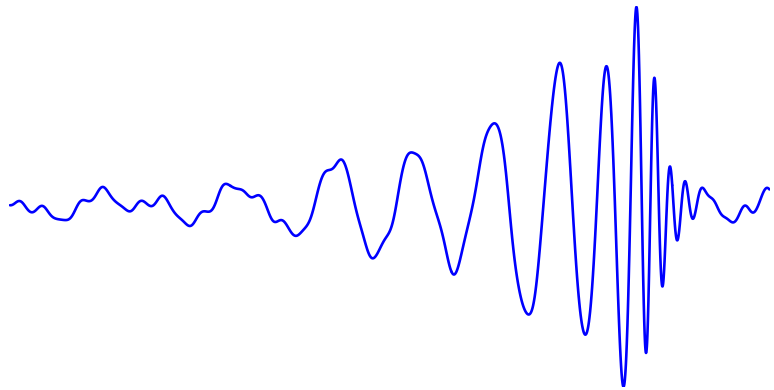

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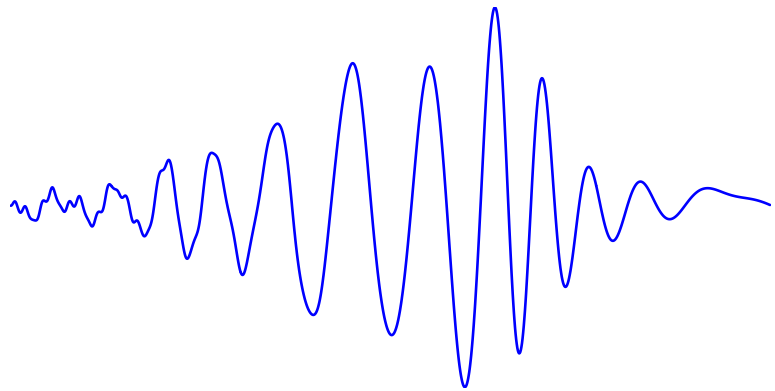

? ? ?

Perhaps more likely a local random tremble. . .

{Effect $\rightarrow$ cause} $\rightarrow$ 'concomitant causes' $\rightarrow$ corroboration

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- We **cannot** say (at least for the very first event) to have observed a gravitational wave, and then we search for the phenomenon which has produced it.

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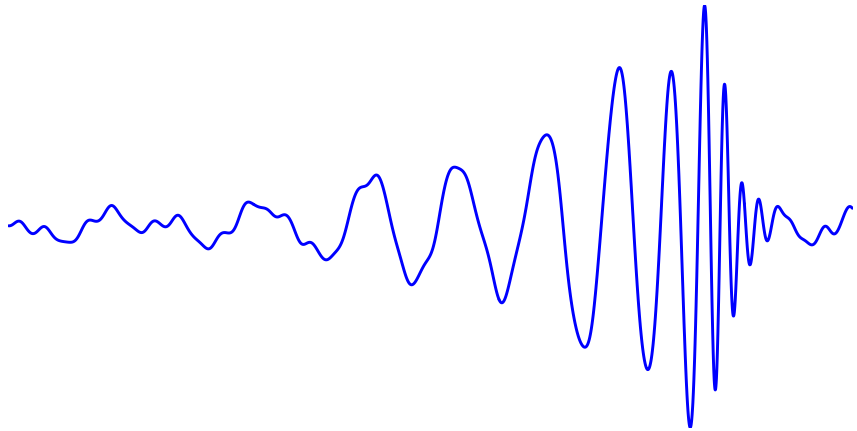
Despite of the 'sigmas'. . .

What is the difference between the two "scribbles"?

A)



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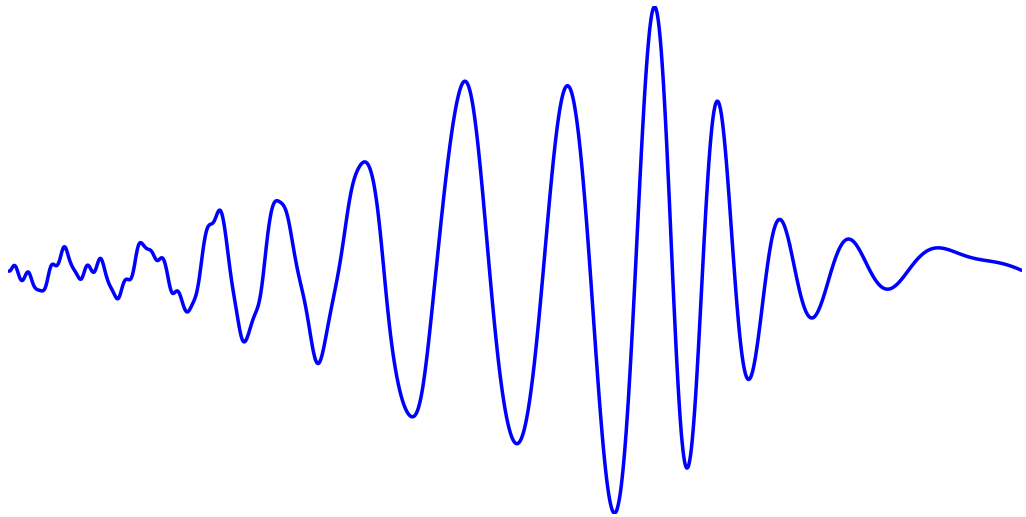

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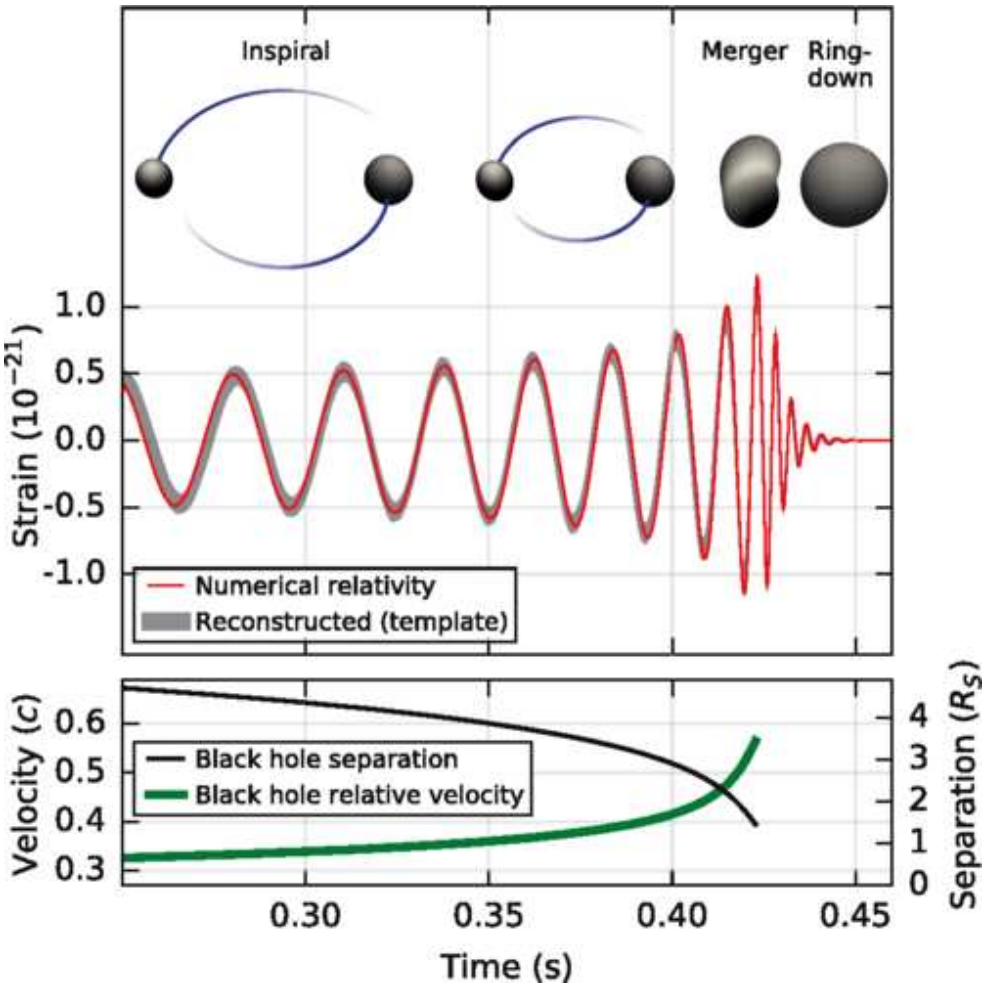

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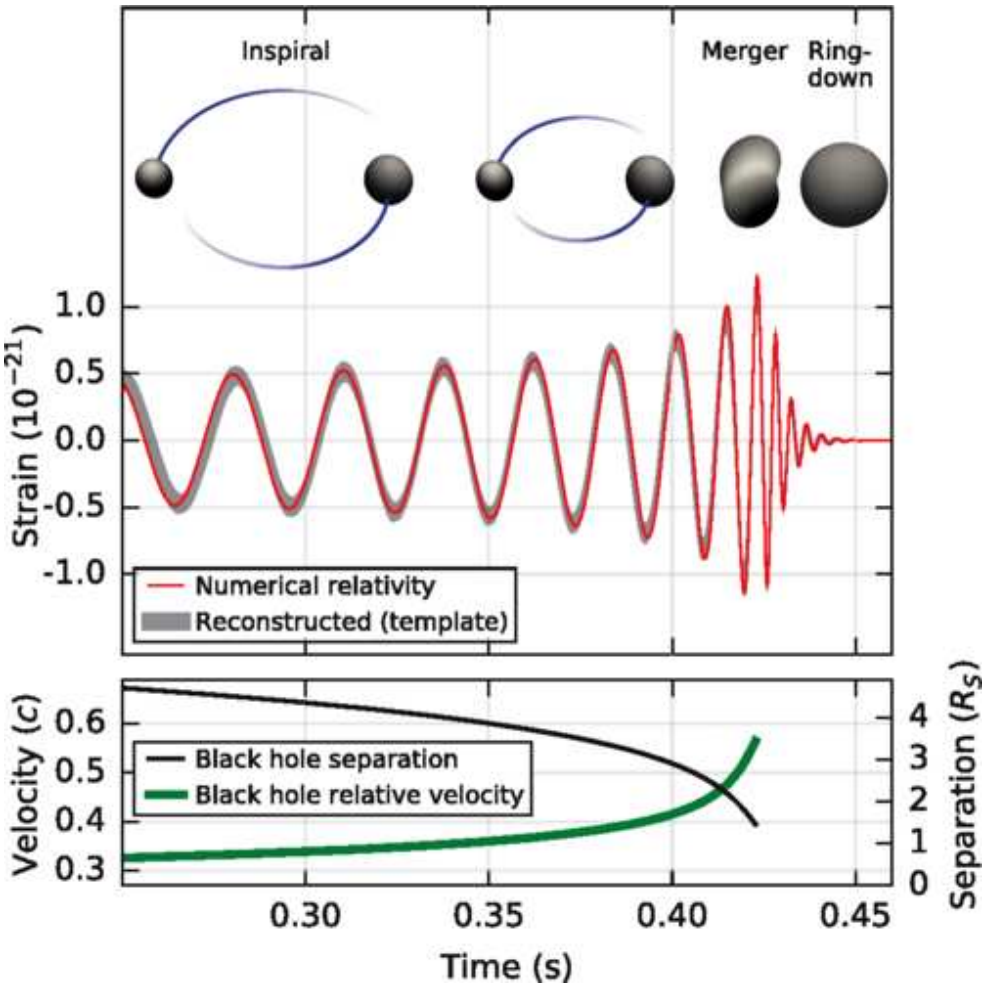

B)

NOTHING (as far as we understand it <u>now</u>...)

From the cosmic space down to problems of common mortals

An example easy to understand:

- two causes;
- two effects;

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From the cosmic space down to problems of common mortals

An example easy to understand:

- two causes;
- two effects;
- medical diagnostics helps to clarify the issues:
  - easier to reach intuitive answers
  - . . . although if someone might have fallacious intuitions
    $\Rightarrow$ a formal guide helps us avoiding errors
    $\Rightarrow$ **logics of the uncertain** (theory of probabilities)

AIDS test

An Italian citizen is selected at random
to undergo an AIDS test.
$\rightarrow$ Performance of clinical trial is not perfect, as customary:

$$
\begin{aligned}
P(\text{Pos} \,|\, \text{HIV}) &= 100\% \\
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\end{aligned}
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H_1='HIV' (Infected) $\qquad$ E_1 = Positive

H_2='$\overline{\text{HIV}}$' (Healthy) $\qquad$ E_2 = Negative

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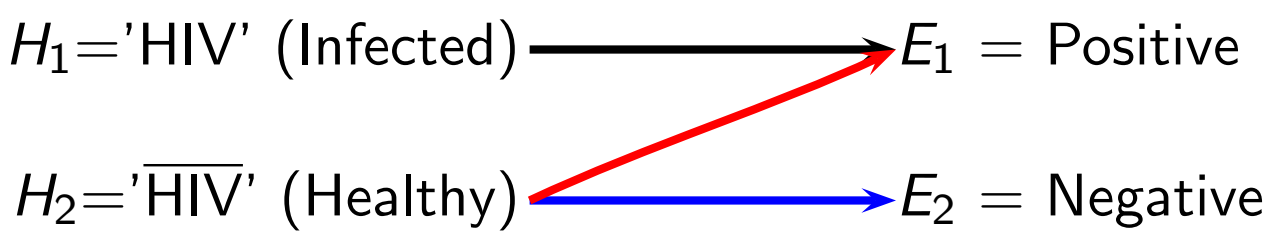

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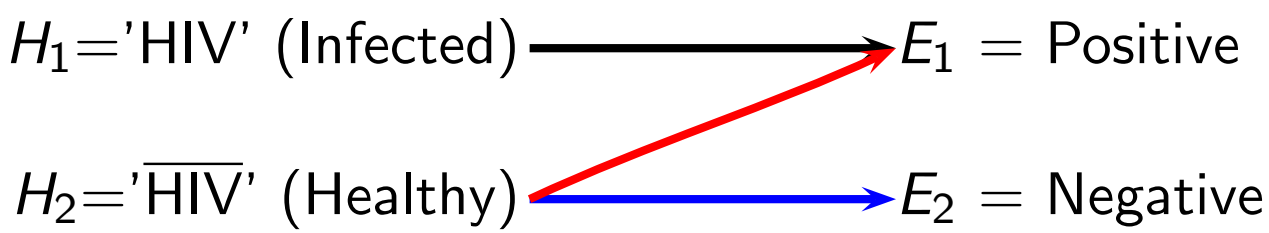

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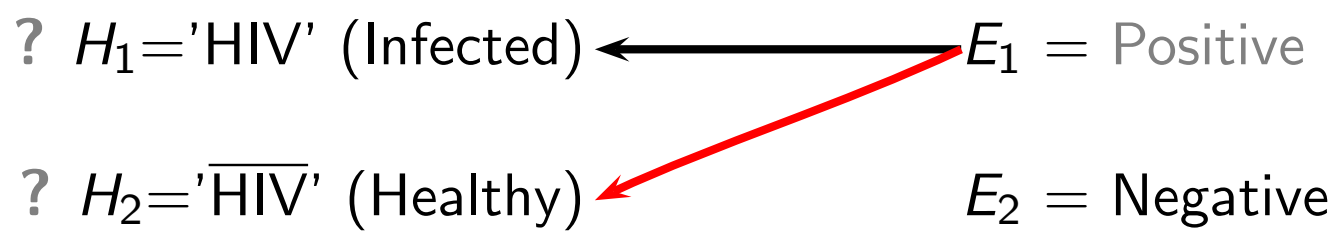

Result: $\Rightarrow$ Positive
Infected or healthy?

AIDS test: how to interpret the result?

Being $P(\text{Pos} \,|\, \overline{\text{HIV}}) = 0.2\%$ and having observed 'Positive', can we say?

- "It is practically impossible that the person is healthy, since it was practically impossible that an healthy person would result positive"

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Instead, $P(\text{HIV}\,|\,\text{Pos, random Italian}) \approx 45\%$

(We will learn in the sequel how to evaluate it correctly)

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NO

Instead, $P(\text{HIV}\,|\,\text{Pos, random Italian}) \approx 45\%$

$\Rightarrow$ Serious mistake! (not just 99.8% instead of 98.3% or so)

AIDS test

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The previous statements, although dealing with probabilistic issues, **are not ground** on probability theory

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$\Rightarrow$ A sound formal guidance can rescue us

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$$\text{In general } P(A \mid B) \neq P(B \mid A)$$

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$\Rightarrow$ Prosecutor's fallacy

Most events 'had' very small probability to occur!

A practical example:

- I shut a picture with my faithful pocket camera.

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What else?

An so on...

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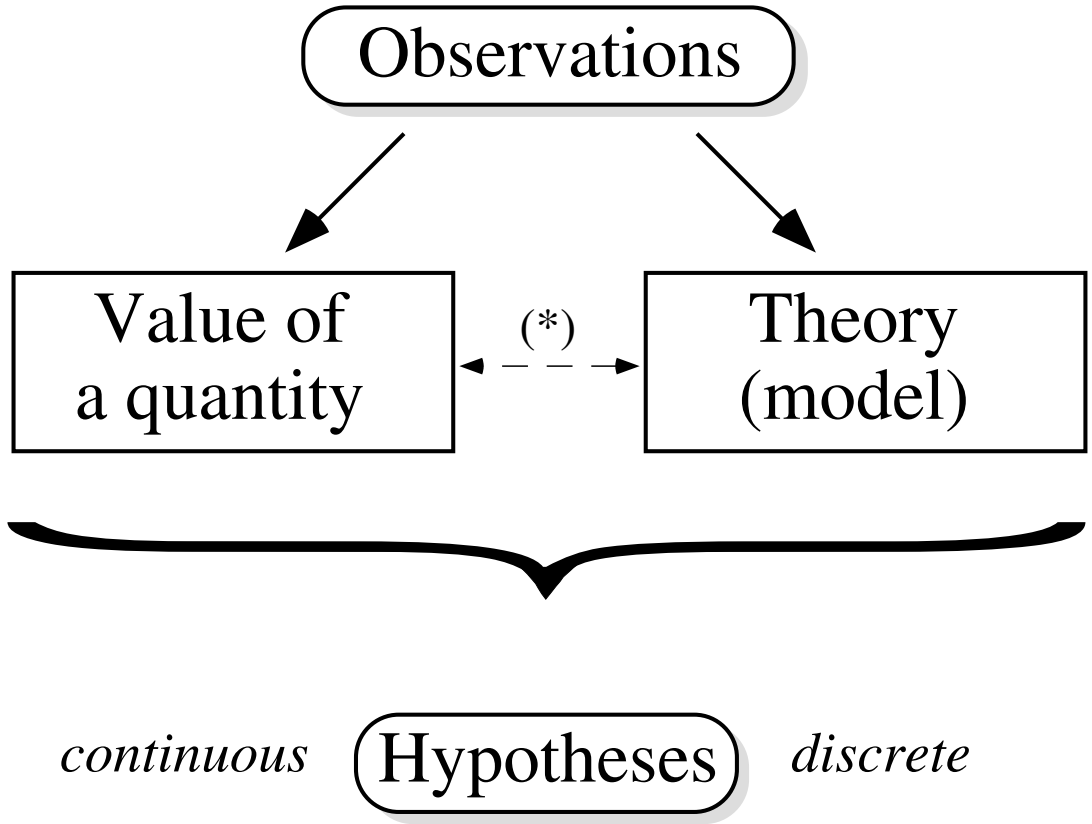
Learning from data



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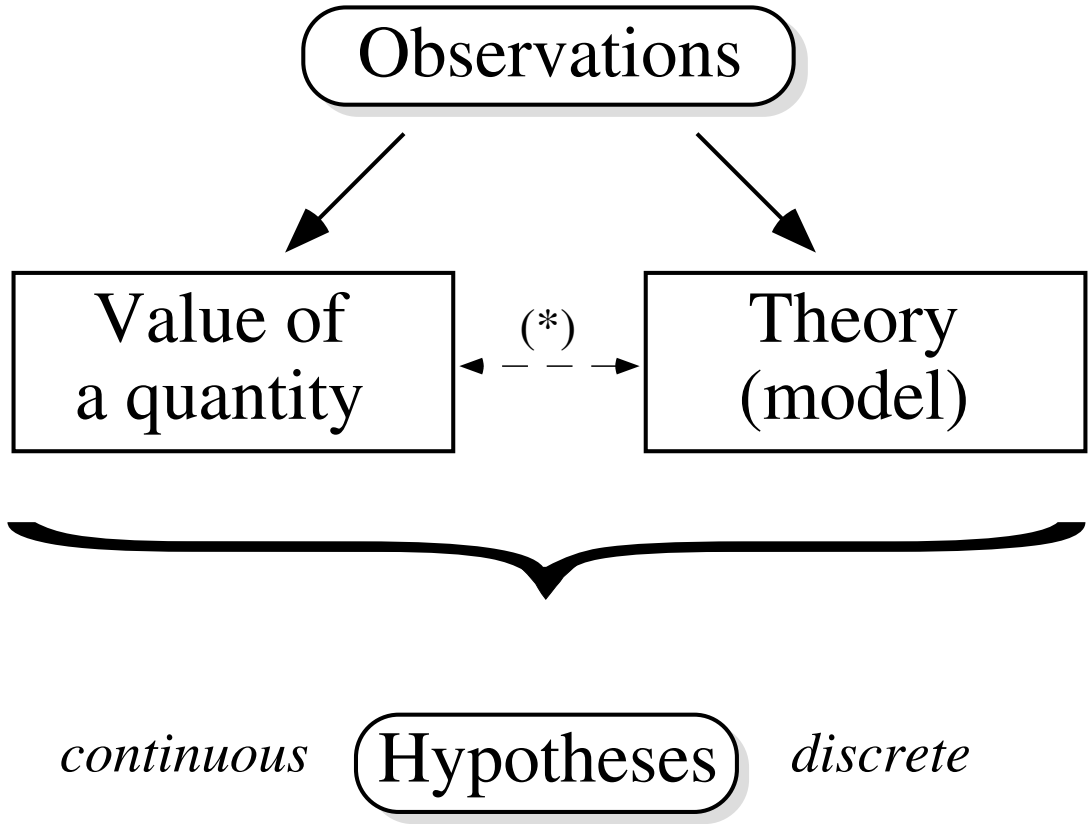
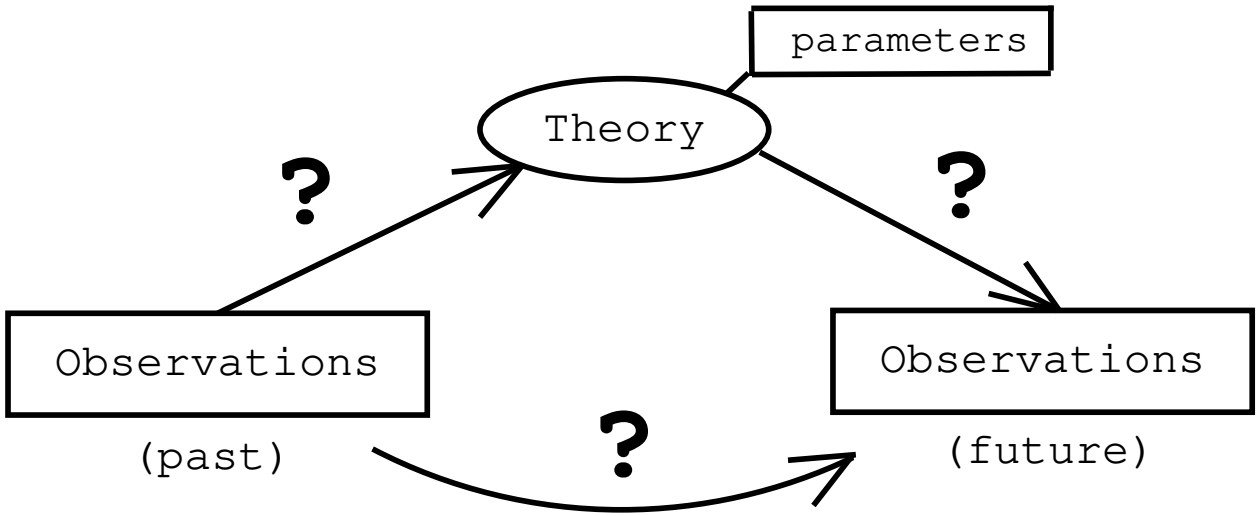


(*) A quantity might be meaningful only within a theory/model

From past to future



Our task:

- Describe/understand the 'physical world'

  $\Rightarrow$ inference of laws ('models') and their parameters $[\Theta]$

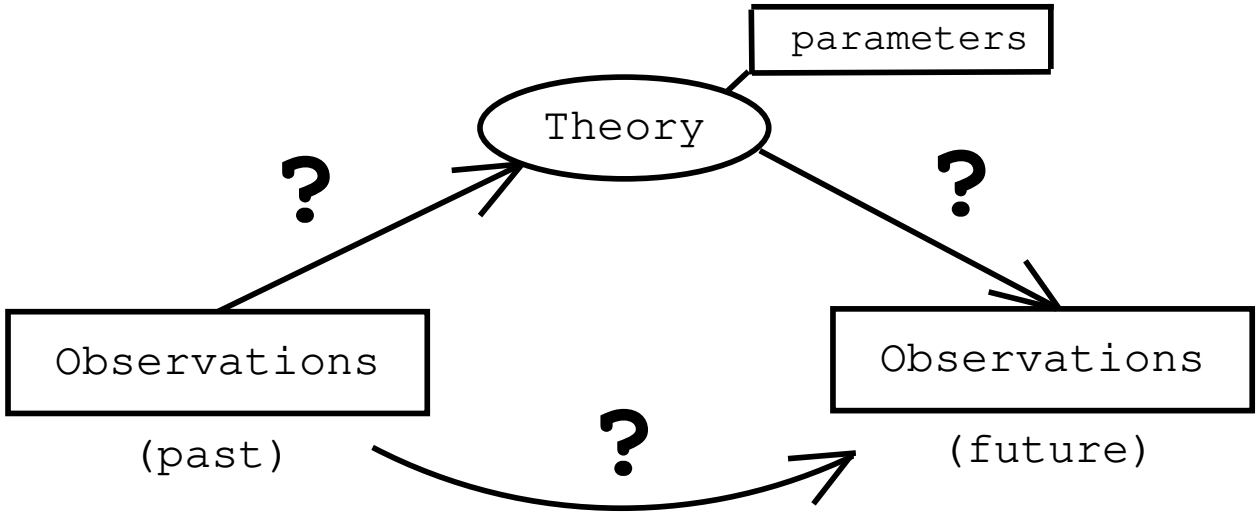
  $\Rightarrow$ $[\Theta \,|\, X_{past}]$

- Predict observations $[X]$

  $\Rightarrow$ forecasting

  $\Rightarrow$ $[X_{future} \,|\, \Theta] \quad \rightarrow \quad [X_{future} \,|\, X_{past}]$

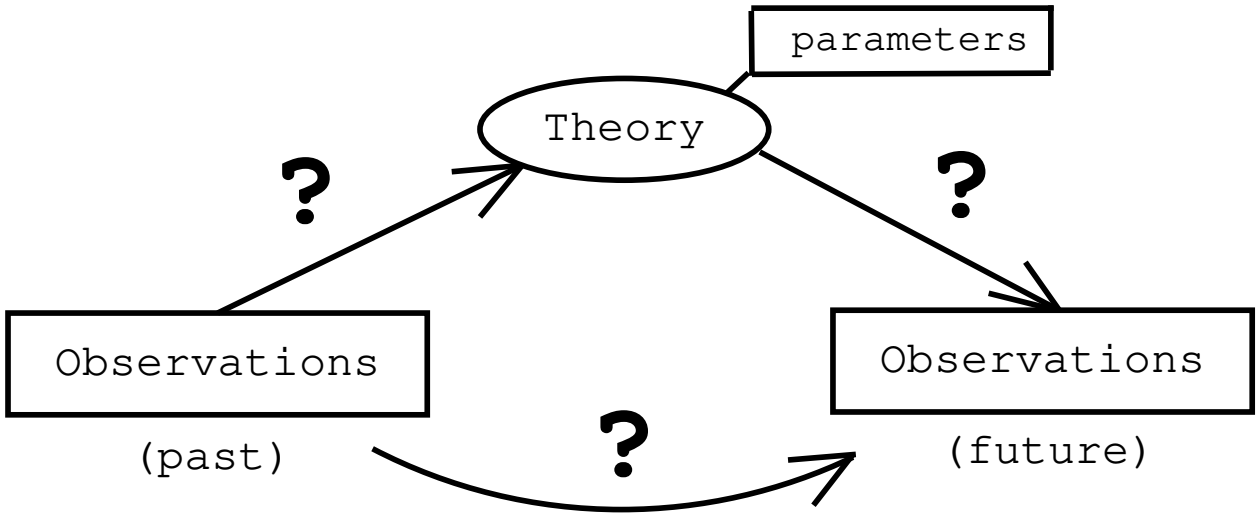
From past to future



Process

- neither automatic
- nor purely contemplative
  - → 'scientific method'
  - → planned experiments ('actions') ⇒ decision.

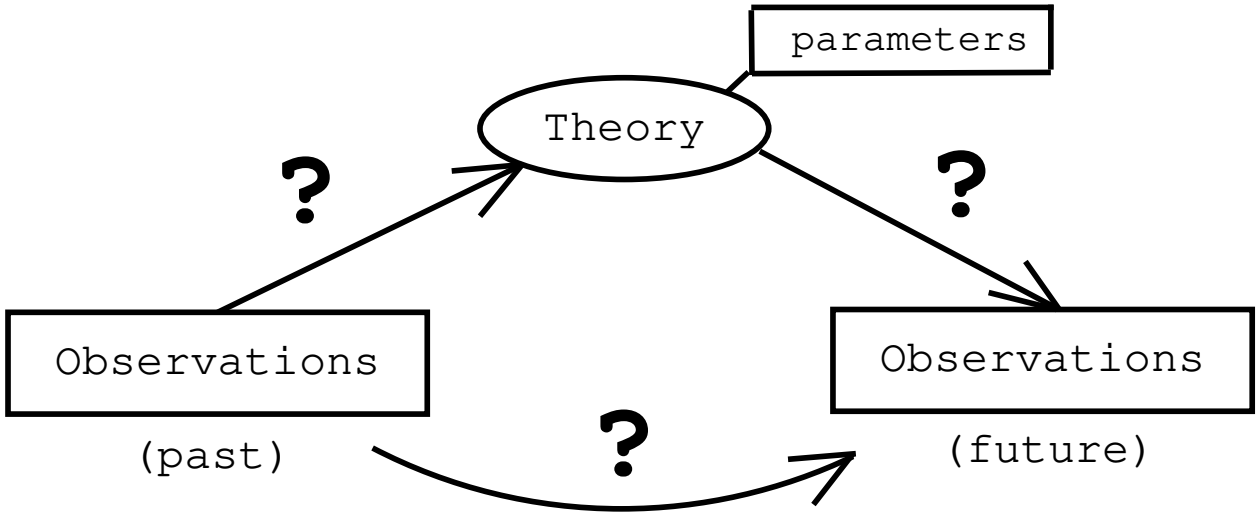
From past to future



$\Rightarrow$ The role of theories/models

- a theory and its parameters are the 'distillate' of all our knowledge about the 'universe' of interest;
- empirical analogical thinking is in most cases not usable:
  - A theory can predict effects never observed before
  - Example: shooting a bullet

From past to future



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"La cognizione d'un solo effetto acquistata per le sue cause ci apre l'intelletto a 'ntendere ed assicurarci d'altri effetti senza bisogno di ricorrere alle esperienze" (Galileo)

Model thinking

"The scientific method is based on repeated experiments"
(or some like that)

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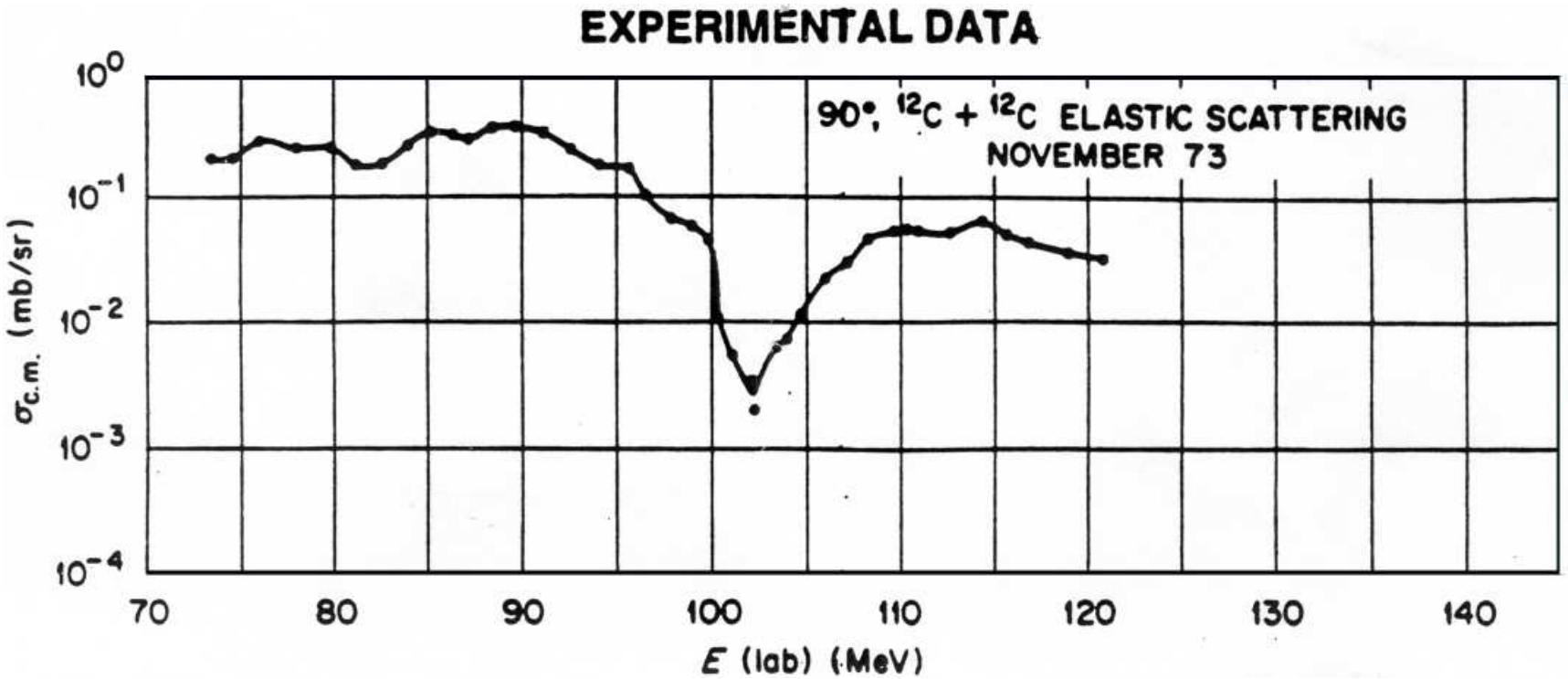
What really matters is to have a Model which links parameters to observations
But remind that "all models are wrong, some are useful"...

Inferential-predictive process



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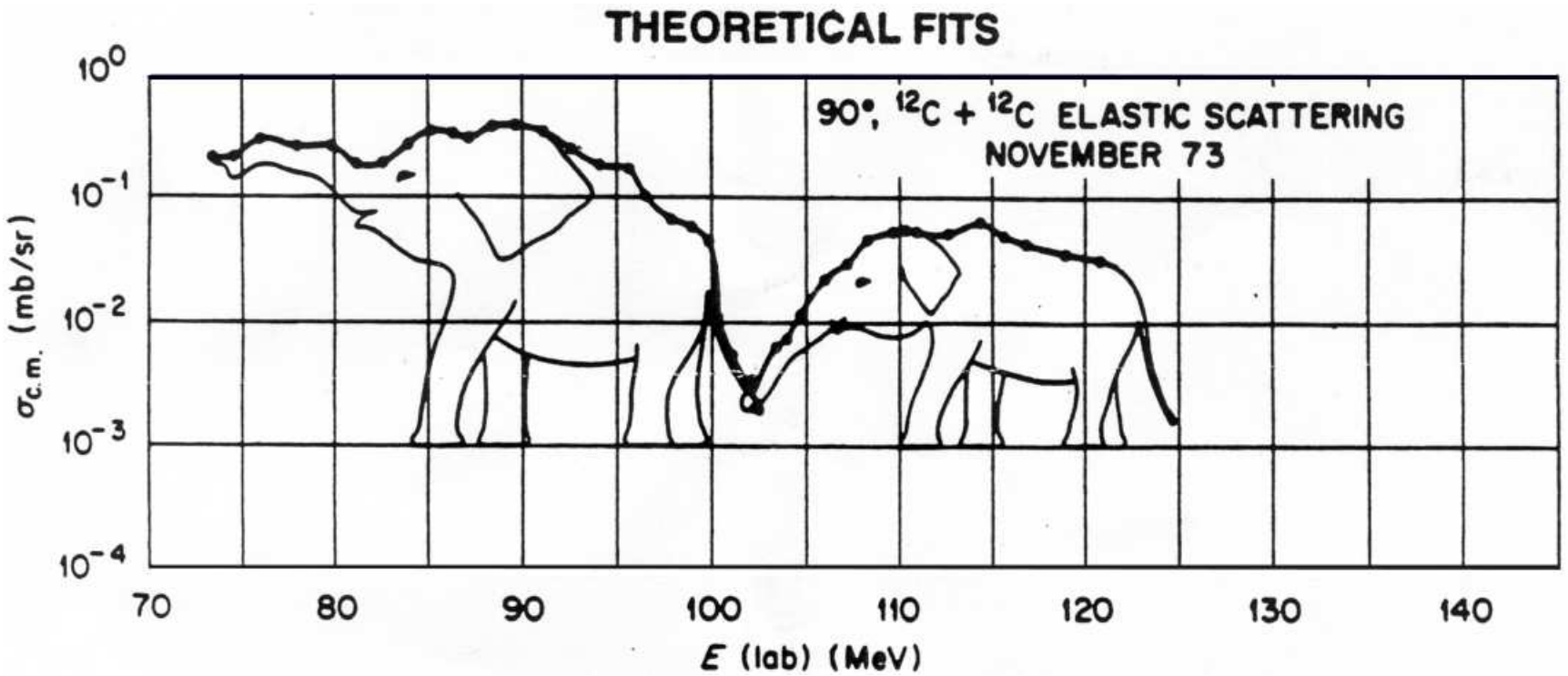

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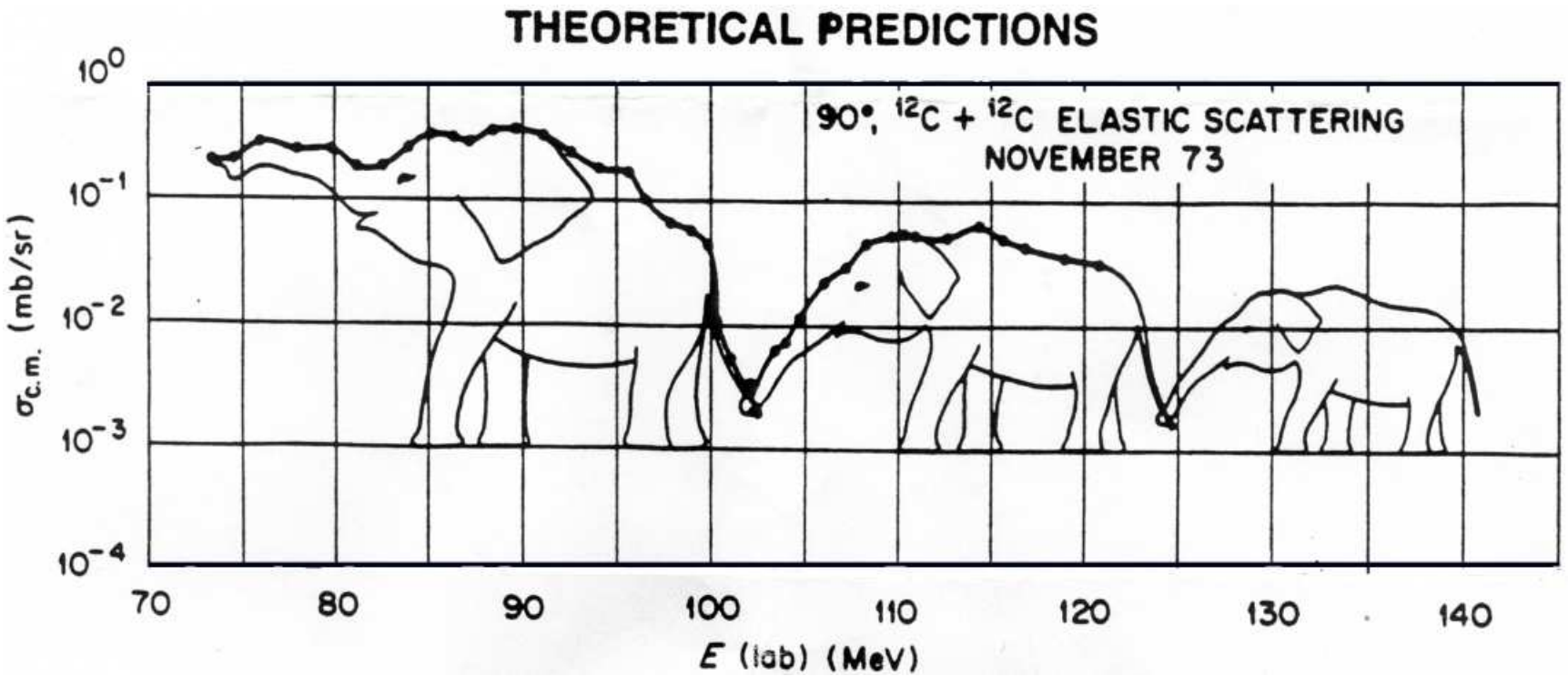
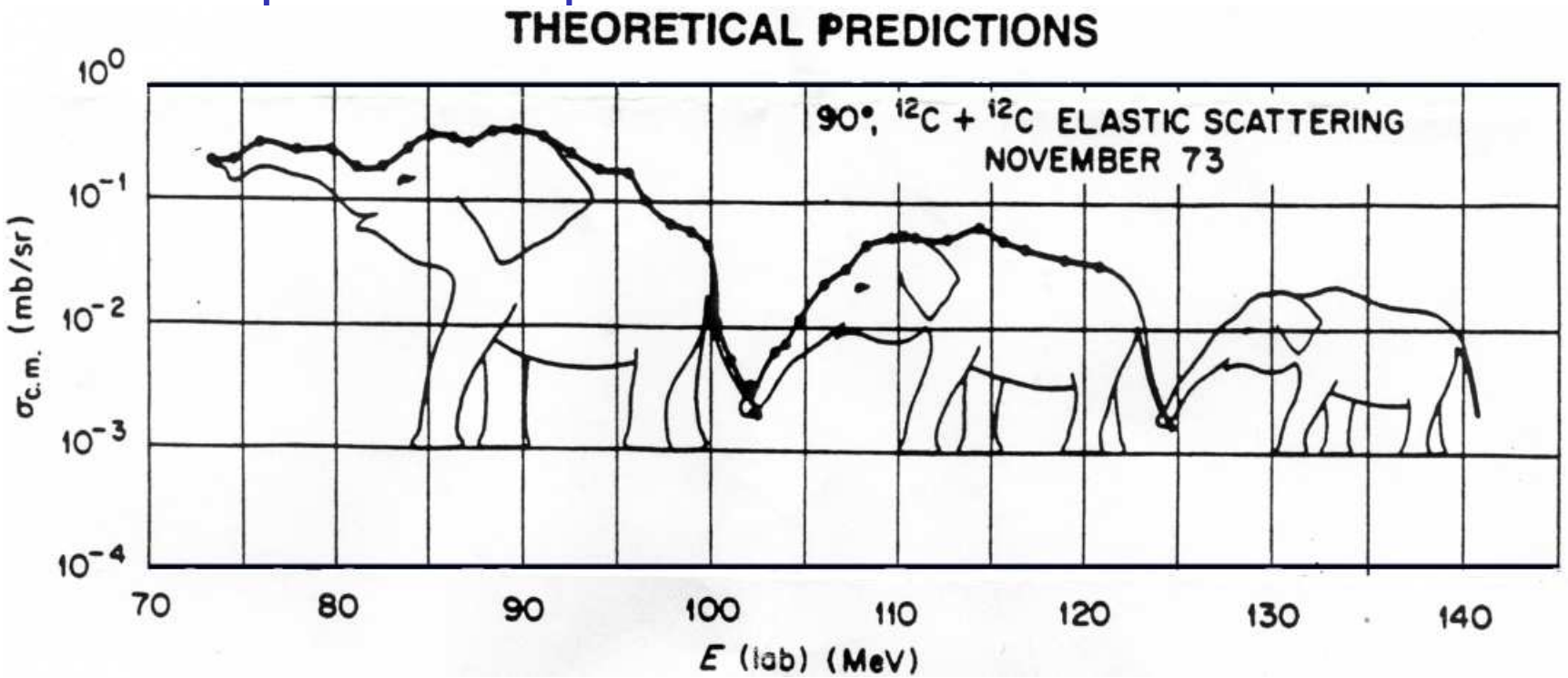


(S. Raman, *Science with a smile*)

Inferential-predictive process



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Even if the (*ad hoc*) model fits perfectly the data,
we do not believe the predictions
because we don't trust the model!

[Many 'good' models are *ad hoc* models!]

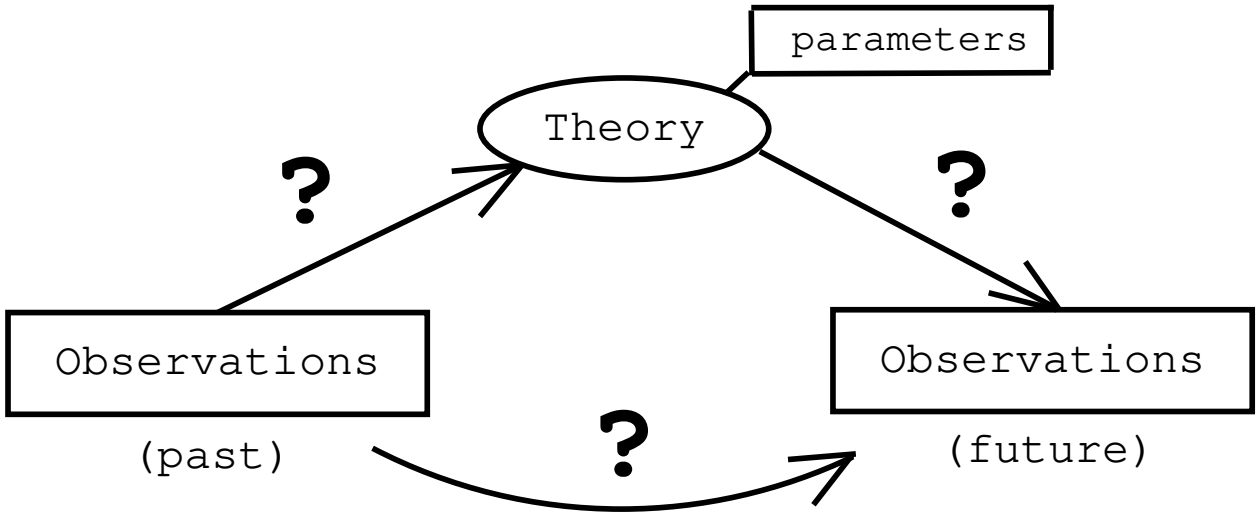
2011 IgNobel prize in Mathematics

- D. Martin of USA (who predicted the world would end in 1954)
- P. Robertson of USA (who predicted the world would end in 1982)
- E. Clare Prophet of the USA (who predicted the world would end in 1990)
- L.J. Rim of KOREA (who predicted the world would end in 1992)
- C. Mwerinde of UGANDA (who predicted the world would end in 1999)
- H. Camping of the USA (who predicted the world would end on September 6, 1994 and later predicted that the world will end on October 21, 2011)

2011 IgNobel prize in Mathematics

"For teaching the world to be careful when making mathematical assumptions and calculations"

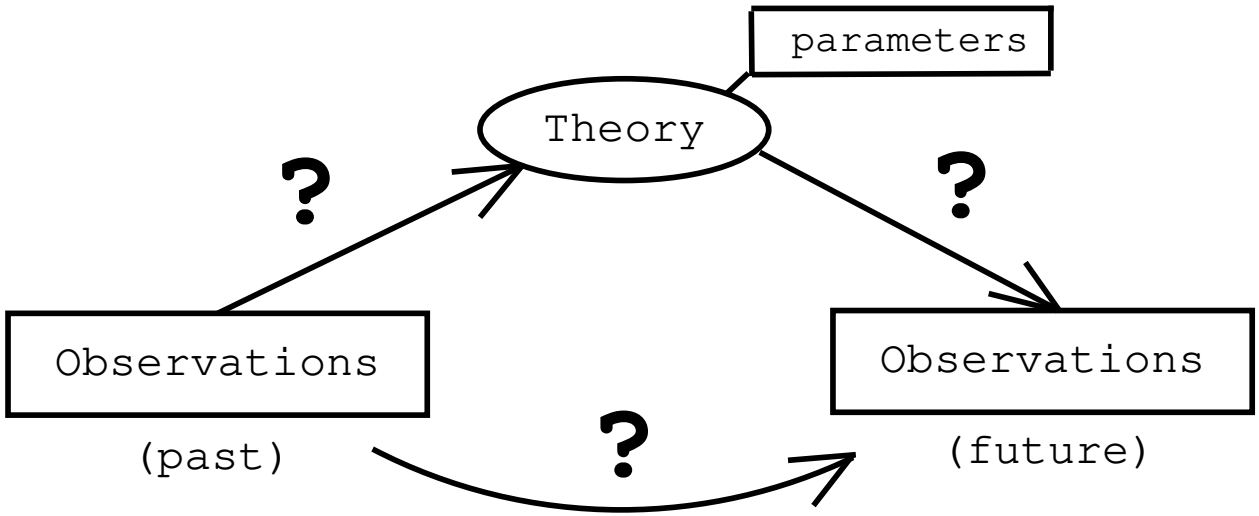
Uncertainty



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the parameters of the model (and/or the model itself)
2. Even if we were sure about theory and parameters, there could be internal ("noise", variables out of our control) or external effects (initial/boundary conditions, 'errors', etc) that make the forecasting uncertain.

Uncertainty

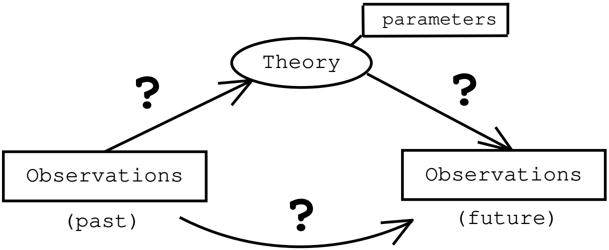


$\Rightarrow$ Uncertainty:

- No certainties, only probabilities

- $P(\Theta \mid X_{past})$
- $P(X_{future} \mid \Theta)$
- $P(X_{future} \mid X_{past})$

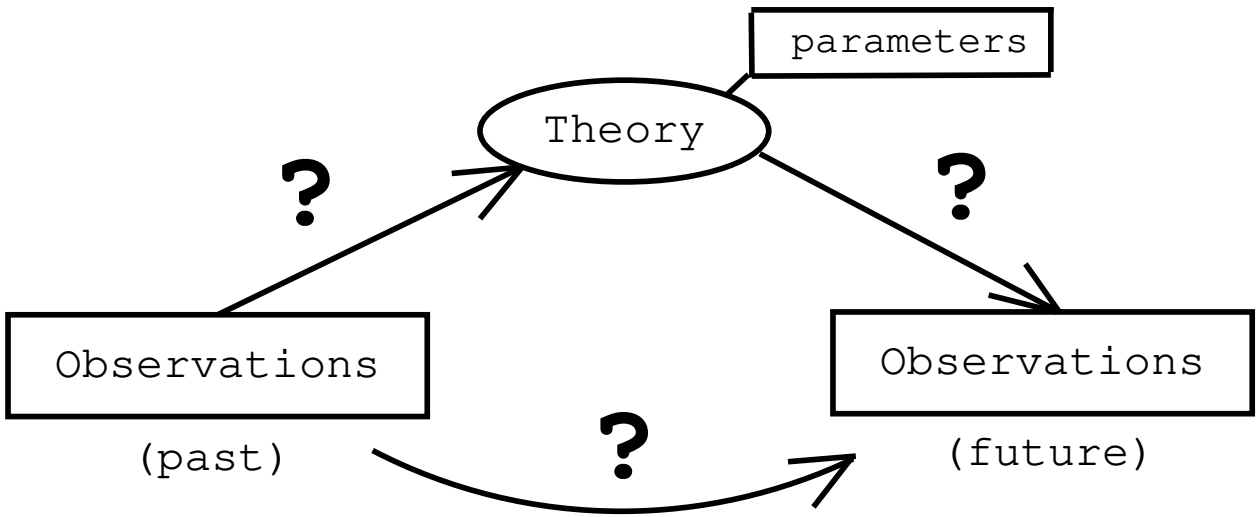
Deep source of uncertainty



Uncertainty:

Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

Deep source of uncertainty



Uncertainty:

$$\text{Theory} \;\; -?\longrightarrow \;\; \text{Future observations}$$
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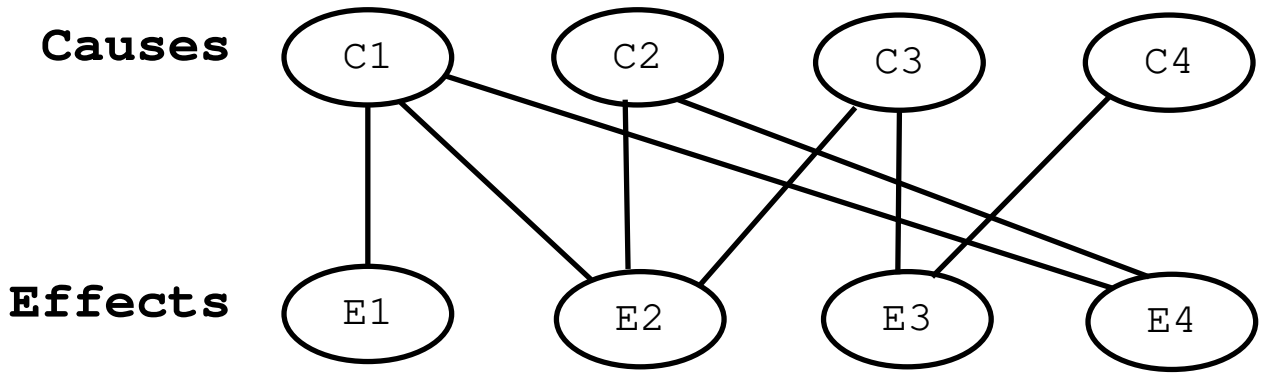
$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ EFFECT

Causes → effects

The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

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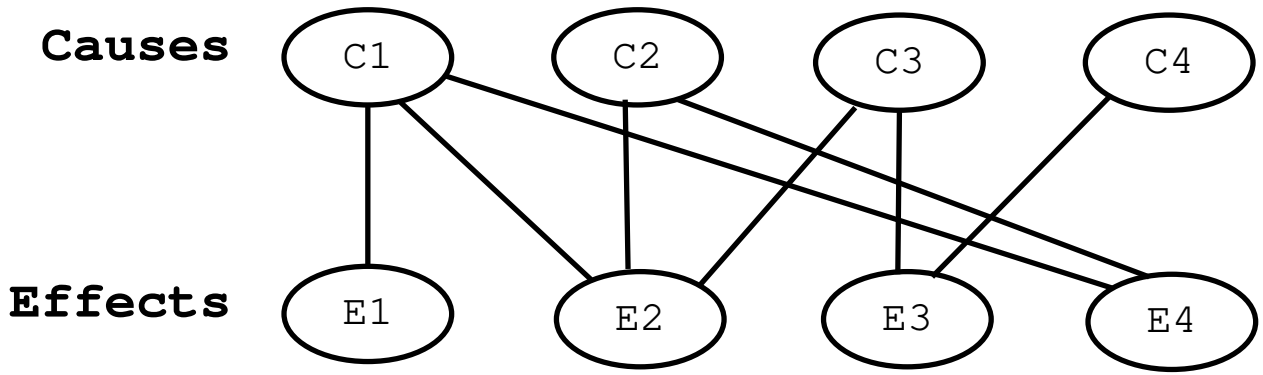

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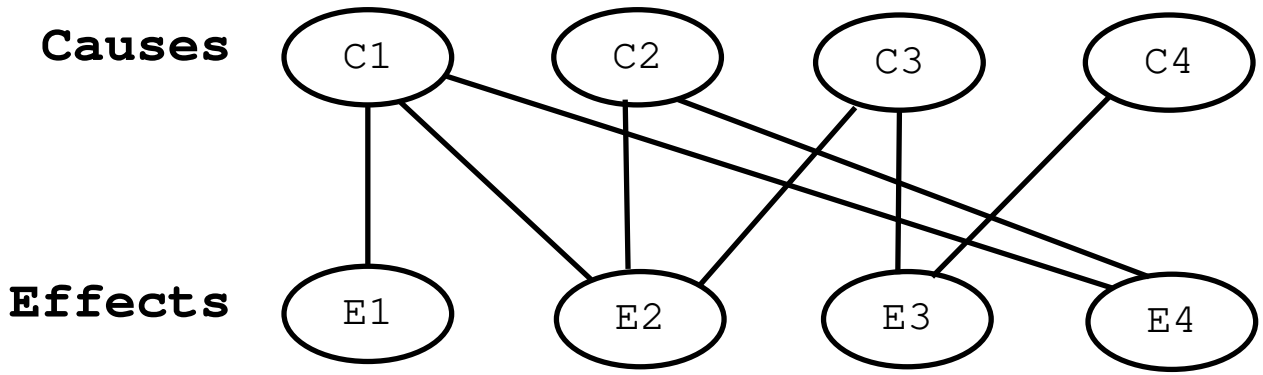


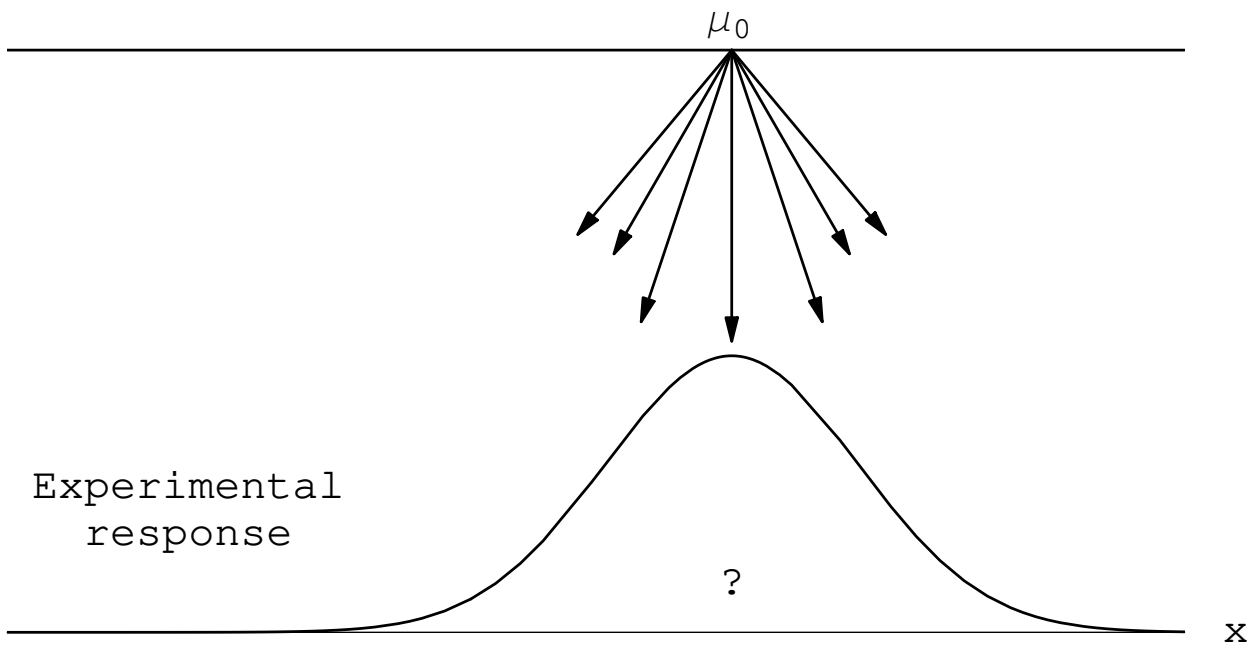
Given an observed effect, we are not sure about the exact cause that has produced it.

$$\mathbf{E_2} \Rightarrow \{C_1,\ C_2,\ C_3\}?$$

→ Probability of causes

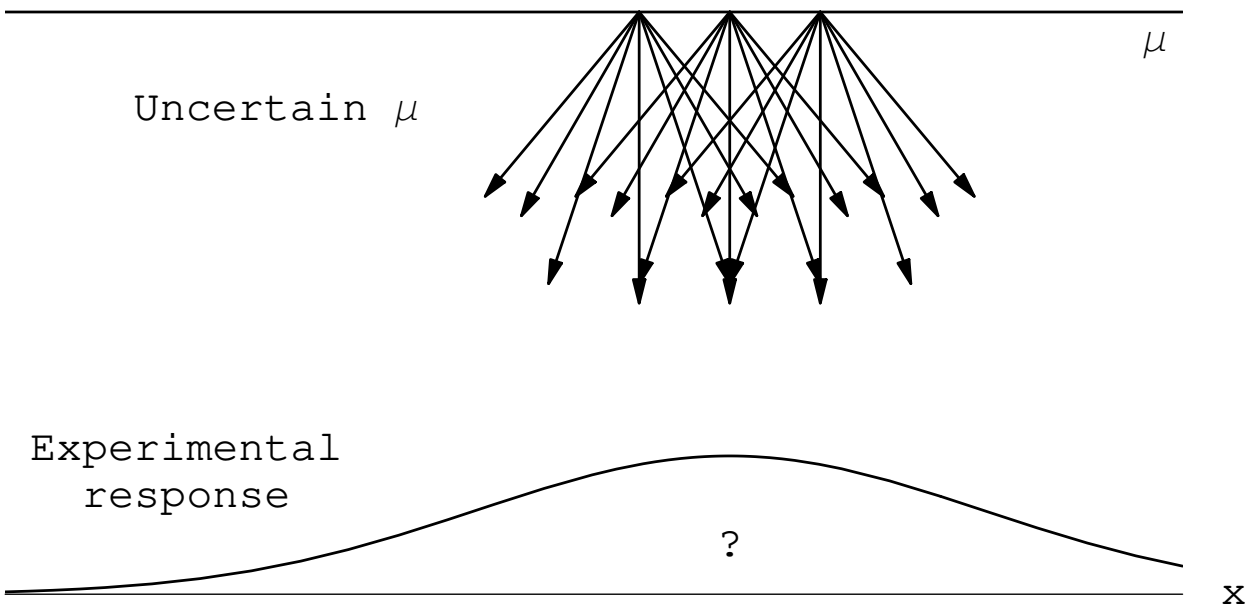
"the essential problem of the experimental method"

From ‘true value’ to observations



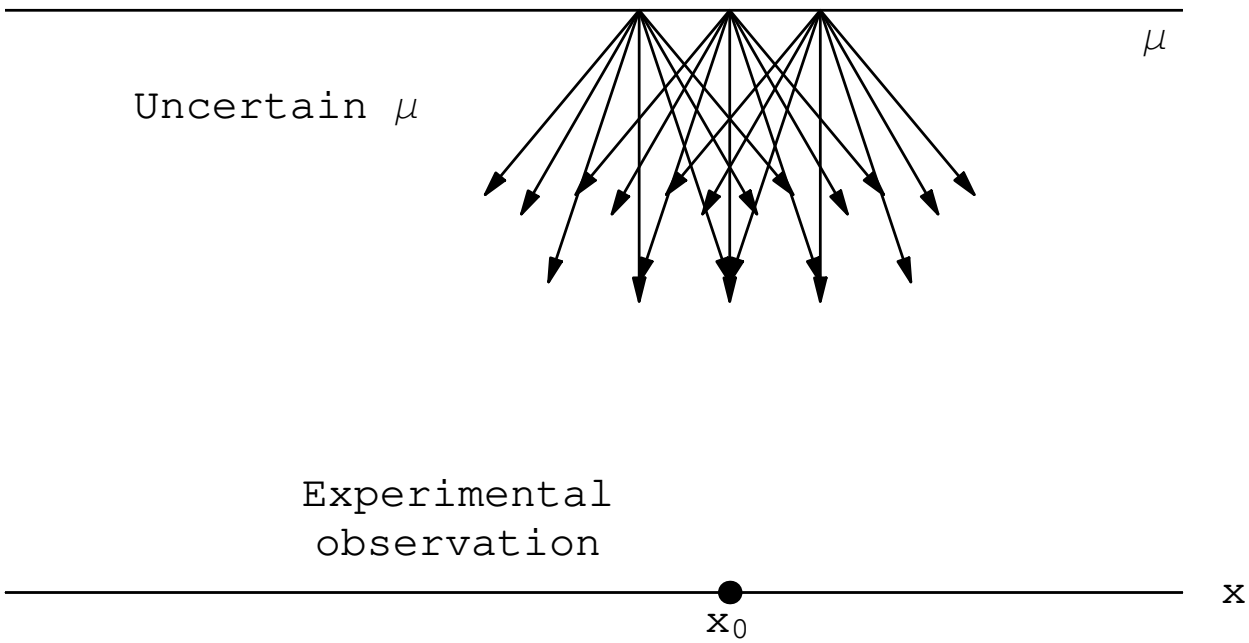
Given μ (exactly known) we are uncertain about x

From ‘true value’ to observations



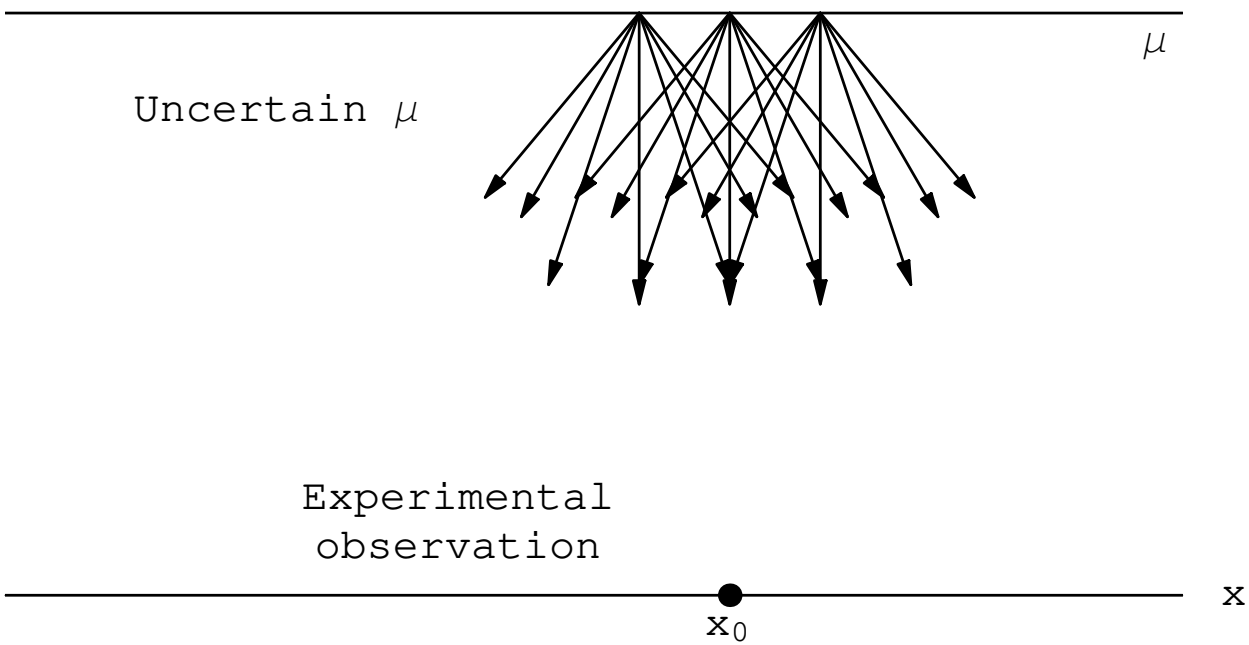
Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



The observed data is <u>certain</u>: $\rightarrow$ 'true value' uncertain.

. . . and back: Inferring a true value



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"data uncertainty" ?

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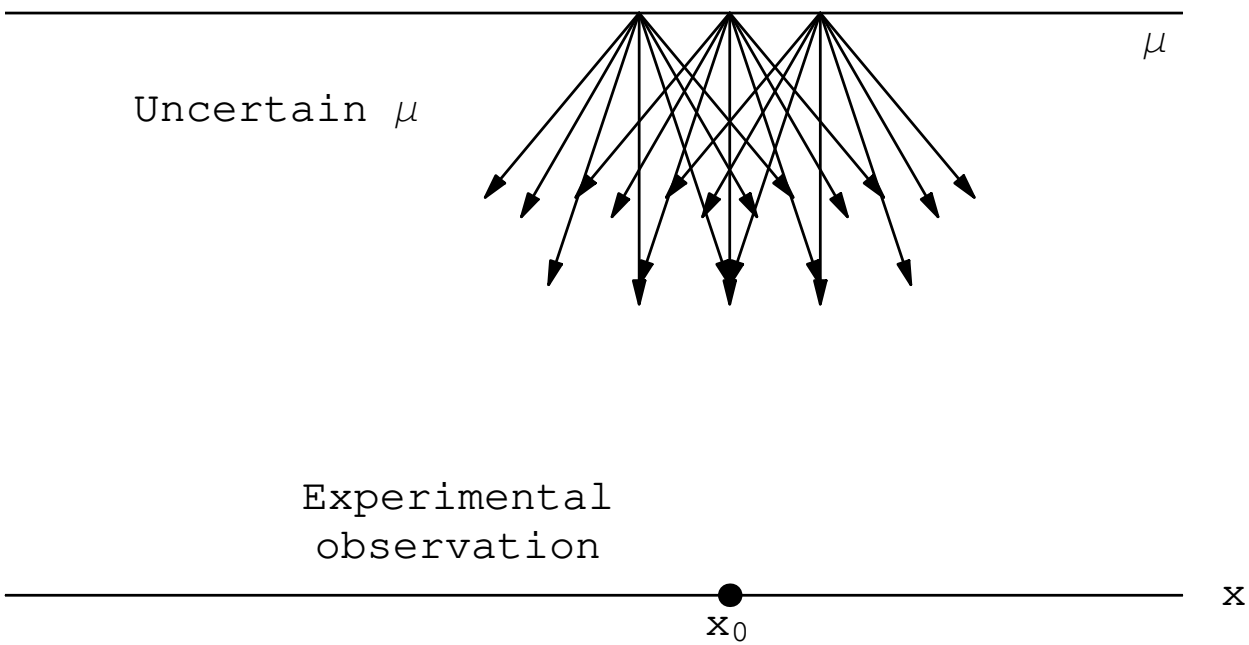

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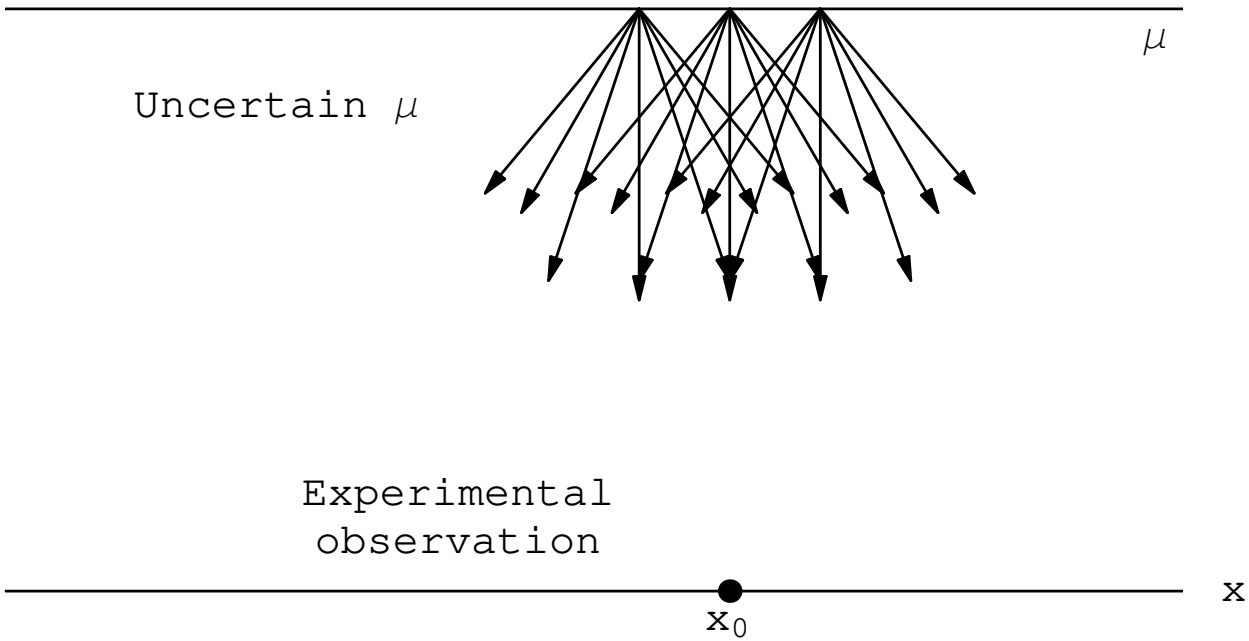

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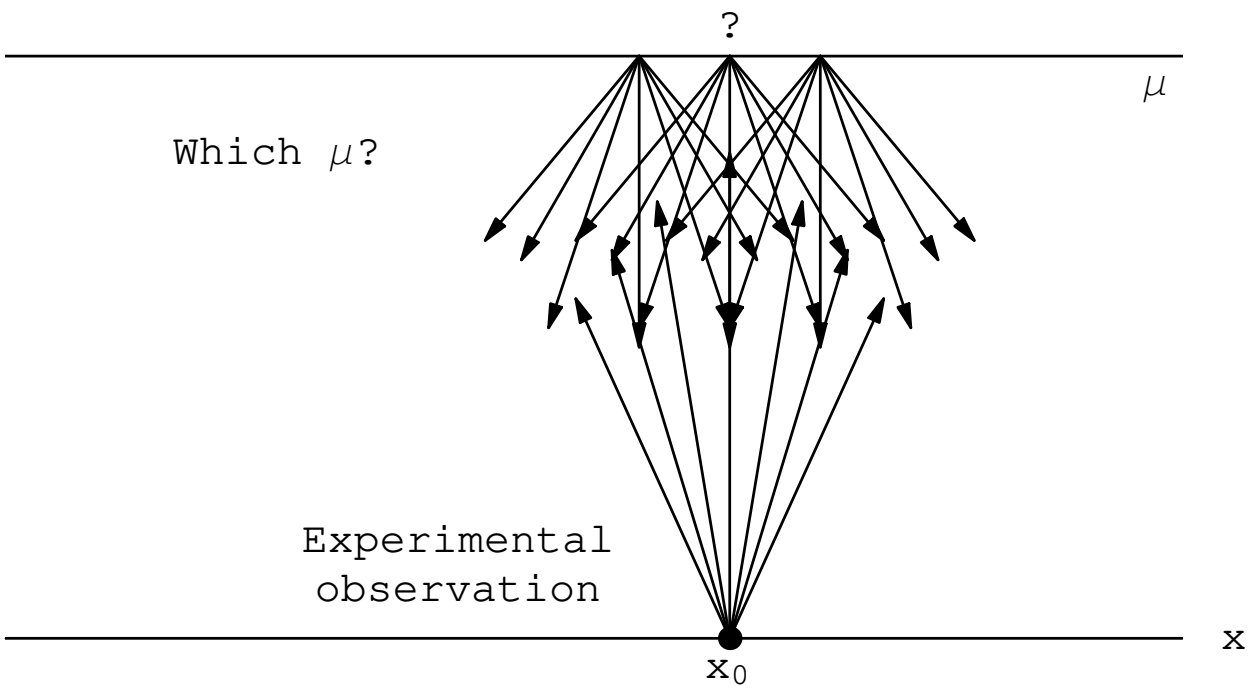
"data uncertainty" ? Data corrupted?

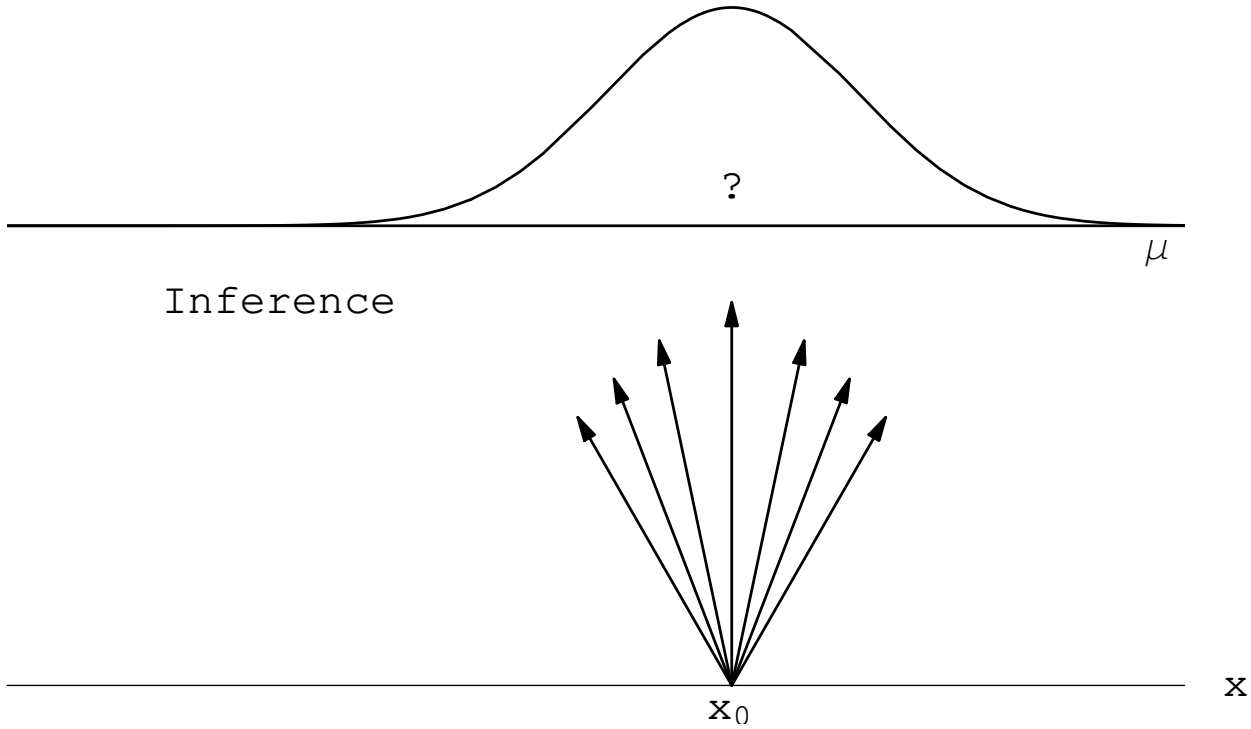
Even if the data were corrupted, the _data_ were the corrupted data**!!** . . .

. . . and back: Inferring a true value



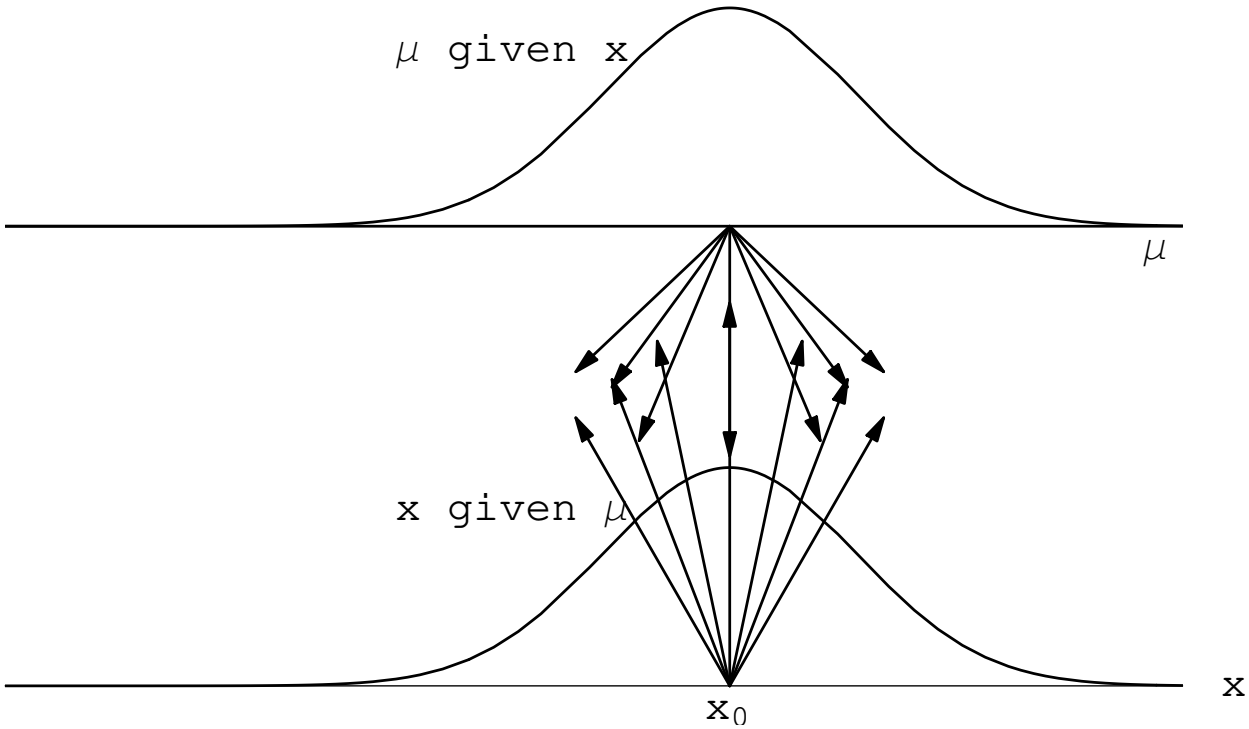
Where does the observed value of x comes from?

. . . and back: Inferring a true value



We are now uncertain about μ, given x.

. . . and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

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- Here
- Now

A very simple experiment

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For simplicity

- μ can assume only six possibilities:

$$\mathbf{0}, \mathbf{1}, \ldots, \mathbf{5}$$

- x is binary:

$$\mathbf{0}, \mathbf{1}$$

$$[\,(1,2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots\,]$$

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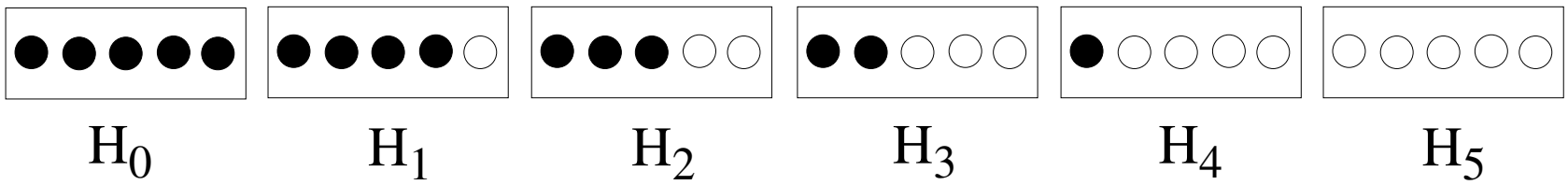
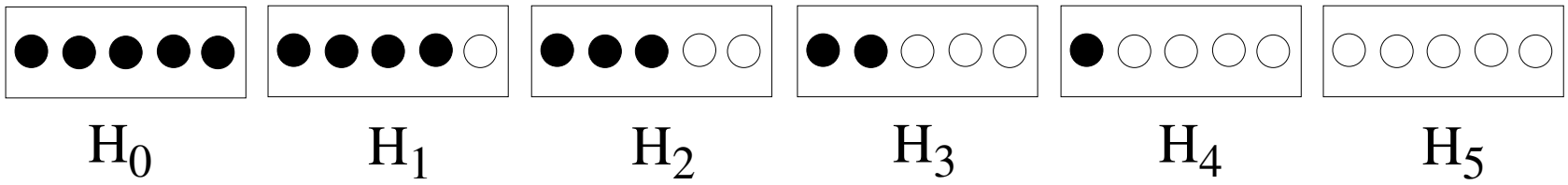
$\Rightarrow$ Later we shall make μ continuous.

Which box? Which ball?



Let us take randomly one of the boxes.

Which box? Which ball?



Let us take randomly one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

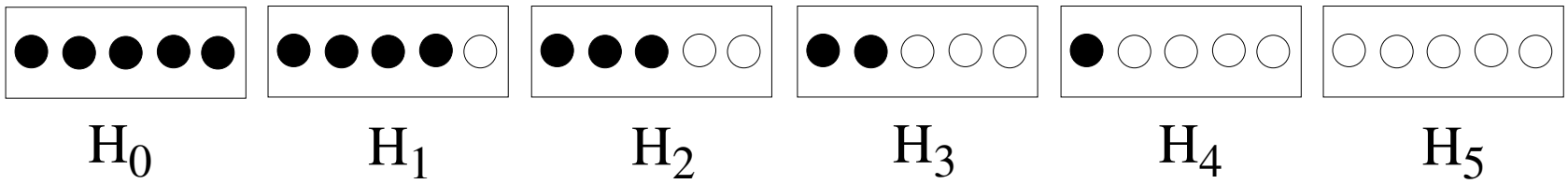
(a) Which box have we chosen, H_0, H_1, . . . , H_5?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\cup_{j=0}^{5} H_j = \Omega$$
$$\cup_{i=1}^{2} E_i = \Omega\,.$$

Which box? Which ball?



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- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

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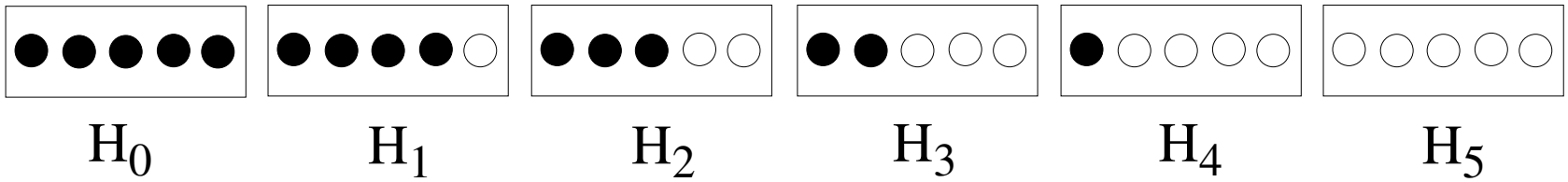

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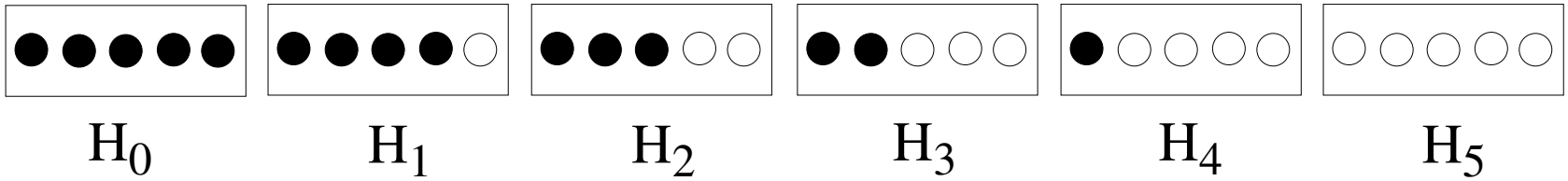

Let us take randomly one of the boxes.

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  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
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  - Can we do it *quantitatively*, in an 'objective way'?
- And after a sequence of extractions?

The toy inferential experiment

The aim of the experiment will be to guess the content of the box without looking inside it, only extracting a ball, record its color and reintroducing in the box

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This toy experiment is conceptually very close to what we do in the pure and applied sciences

$\Rightarrow$ try to guess what we cannot see (the electron mass, a magnetic field, etc)

. . . from what we can see (somehow) with our senses.

The rule of the game is that we are not allowed to watch inside the box! (As we cannot open and electron and read its properties, unlike we read the MAC address of a PC interface.)

Where is probability?

We all agree that the experimental results change

- the probabilities of the box compositions;
- the probabilities of a future outcomes,

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Certainly not *in* the box!

Subjective nature of probability

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Probability depends on the status of information of the *subject* who evaluates it.

Probability is always conditional probability

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where $I_s(t)$ is the information available to *subject* s at time t.

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Examples:

- tossing coins and dice;
- the three box problem.

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$\Rightarrow$ How much we believe something

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$\rightarrow$ 'Degree of belief' $\leftarrow$

y

The End