Bayesian Reasoning in High Energy Physics
- Principles and Applications -

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Abstract

Bayesian statistics is based on the intuitive idea that probability quantifies the degree of belief in the occurrence of an event. The choice of name is due to the key role played by Bayes' theorem, as a logical tool to update probability in the light of new pieces of information. This approach is very close to the intuitive reasoning of experienced physicists, and it allows all kinds of uncertainties to be handled in a consistent way. Many cases of evaluation of measurement uncertainty are considered in detail in this report, including uncertainty arising from systematic errors, upper/lower limits and unfolding. Approximate methods, very useful in routine applications, are provided and several standard methods are recovered for cases in which the (often hidden) assumptions on which they are based hold.

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Introduction

These notes are based on seminars and minicourses given in various places over the last four years. In particular, lectures I gave to graduate students in Rome and to summer students in DESY in the spring and summer of 1995 encouraged me to write the 'Bayesian primer', which still forms the core of this script. I took advantage of the academic training given at CERN at the end of May 1998 to add some material developed in the meantime.

Instead of completely rewriting the primer, producing a thicker report which would have been harder to read sequentially, I have divided the text into three parts.

- The first part is dedicated to a critical review of standard statistical methods and to a general overview of the proposed alternative. It contains references to the other two parts for details.
- The second part essentially reproduces the old primer, subdivided into chapters for easier reading and with some small corrections.
- Part three contains an appendix, covering remarks on the general aspects of probability, as well as other applications.

The advantage of this structure is that the reader can have an overall view of problems and proposed solutions and then decide if he wants to enter into details.

This structure inevitably leads to some repetition, which I have tried to keep to a minimum. In any case, *repetita juvant*, especially in this subject where the real difficulty is not understanding the formalism, but shaking off deep-rooted prejudices. This is also the reason why this report is somewhat verbose (I have to admit) and contains a plethora of footnotes, indicating that this topic requires a more extensive treatise.

A last comment concerns the title of the report. As discussed in the last lecture at CERN, a title which was closer to the spirit of the lectures would have been "Probabilistic reasoning ... ". In fact, I think the important thing is to have a theory of uncertainty in which "probability" has the same meaning for everybody: precisely that meaning which the human mind has developed naturally and which frequentists have tried to kill. Using the term "Bayesian" might seem somewhat reductive, as if the methods illustrated here would always require explicit use of Bayes' theorem. However, in common usage 'Bayesian' is a synonym of 'based on subjective probability', and this is the reason why these methods are the most general to handle uncertainty. Therefore, I have left the title of the lectures, with the hope of attracting the attention of those who are curious about what 'Bayesian' might mean.

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